

Some Matrix Notation:

$$\text{If } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

The cofactor of a_{ij} in $A = A_{ij}$

$$A_{ij} = (-1)^{i+j} M_{ij}$$

M_{ij} is the minor obtained by omitting the i^{th} row and j^{th} column from A .

For example, from the above 4 by 4 matrix:

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix}$$

So, for example:

$$A_{12} = (-1)^{1+2} M_{12} = - \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix}$$

The order of A is n , so write A_n , and the cofactor associated with a matrix of this order, denote by $A_{ij}^{(n)}$. Define $A_{ij}^{(1)} = 1$.

So:

$$A_{12}^{(2)} \text{ is such that } n = 2, \text{ so with } A^2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \text{ then:}$$

$$A_{12}^{(2)} = (-1)^{1+2} a_{21} = -a_{21}$$

$$A_{31}^{(3)}, \text{ so } n = 3, \text{ with } A^3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$A_{31}^{(3)} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

So:

$$A_{ij}^{(n)} = \text{cofactor associated with a matrix } A \text{ of } n^{\text{th}} \text{ order determinant.}$$

Determinant of $n \times n$ matrix:

$$D_n = \mathbf{e}_{k_1 k_2 \dots k_n} \prod_{i=1}^n a_{i k_i}$$

Where:

$$\prod_{i=1}^n a_{i k_i} = a_{1 k_1} a_{2 k_2} \dots a_{n k_n}$$