Energy from sun:

Emits 3.9×10^{26}W over sphere radius r_o (radius of earths orbit)

Power density on earth's surface: $\dfrac{3.9\times10^{26}}{4pr_0^2} = 1.4\times10^3 Wm^{-2}$

Labelling:

${}^A_Z X_N$	X:	Chemical symbol
	Z:	Atomic number = $\#p$
	N:	$\#n$
	A:	Atomic mass number = $\#p + \#n = N + Z$

Isotopes: same Z, different A.

e.g. hydrogen ${}^1_1 H_0$ Protium 99.985% of all hydrogen

${}^2_1 H_1$ Deuterium (**d**) 0.015%

- Bound state of $p + n$.

$d + 2.2MeV \rightarrow n + p$ (Takes energy to split)

could be done by gamma ray with $E_g \geq 2.2MeV$

${}^3_1 H_2$ Tritium (**t**)

- Bound state of $p + 2n$.

Requires $E_g \geq 8.48MeV$ this time

Will decay to *He* by beta decay, as unstable.

These energies are the *Binding Energy.*

Binding energy of alpha particle: $Be({}^4He) = 28.3MeV$.

Stable means half life > age of universe (10^{10} years).

Nuclear sizes:

Coulomb potential: $V(r) = \dfrac{1}{4pe_0}\dfrac{Z_1 Z_2}{r}e^2$ for two atoms.

Now, as atoms come closer together, *KE* will convert to *PE*…

Particle stops when $PE = KE$, at a distance r_c, the closest approach.

So: $KE = \dfrac{1}{4pe_0}\dfrac{Z_1 Z_2}{r_c}e^2$, for the example of Rutherford's back-scattering (throw alpha's at a gold target)…

$KE_a = \dfrac{1}{4pe_0}\dfrac{Z_{Au}2}{r_c}e^2$, as KE_a will be known, r_c can be found easily. Also, note that $r_{nuclear} < r_c$, so this gives an upper limit on the radius of an atom.

Fine structure constant:

$$a = \frac{e^2}{4pe_0\hbar c} = \frac{1}{137}$$

And, as $\hbar c = 197.3MeV$ the gives: $\dfrac{e^2}{4pe_0} = \dfrac{197.3}{137}MeV.fm$

Nuclear radius $r = r_0 A^{1/3}$ A = Atomic mass number

$r_0 = 1.2$fm (1fm $= 10^{-15}$m)

Nuclear masses & energies:

$1u = 931.5\,{MeV}/{c^2}$ the atomic mass unit.

The mass of a nucleus is the sum of the masses of the constituent nucleons PLUS the contribution from the nuclear binding energy BE.

Use: $E = mc^2$ to convert between mass/energy.

E.g: $m_d < m_p + m_n$ we get a decrease in the mass of the system, as p+n bind to form d.

I.e:

$d + 2.2MeV \to p + n$ Breakup – requires energy

$n + p \to d + 2.2MeV$ Binding – releases energy

In a mass spectrometer, the radius of trajectory $r = \dfrac{m_A v}{Bq}$, so masses can be found.

Gives m_A to a precision of 1 in 10^6.

Define binding energy as the difference between nucleus and its constituents.

Thus:

$$BE = \{Zm_p + Nm_n - m_N\}c^2$$

But, the mass of the nucleus m_N can be expressed as:

$$m_N c^2 = m_A c^2 - Zm_e c^2 + \sum_{i=1}^{Z} BE(e_i)$$

As electrons are also bound; but their BE is so small we neglect it.

Therefore:

$$BE = \{Zm_p + Nm_n - (m_A - Zm_e)\}c^2$$

Also, note that: $Zm_p + Zm_e = Zm_{{}^1_1H_0}$, hence:

$$\text{Nuclear BE} = \left(Zm_{{}^1_1H_0} + Nm_n - m_A\right)c^2$$

These terms are in atomic mass units – the expression for u above should be used to convert into MeV.

e.g:

Calculate $BE\left({}^{238}_{92}U_{146}\right)$ given: ${}^1_1H_0 = 1.00782u$, ${}^{238}_{92}U_{146} = 238.050785u$, $m_n = 1.008665u$

So, $BE = (92\times 1.007825 + 146\times 1.008665 - 238.050785)u = 1.934210u = 1802MeV$

The BE per nucleon can be found: $\dfrac{BE}{A} = \dfrac{1802}{238} = 7.57MeV$

Main points of curve:

- Nuclear binding energies ~8MeV to within 10% across most nuclei.

- Peaks at ~A=60 – most tightly bound nuclei implies gain energy by climbing up curve

- If fuse together two light nuclei (*A <60*) (going *up* curve from the left)

  e.g. $d + t \rightarrow {}^4He + n$ gives a system with higher BE than ingredients, so energy liberated

  Fusion.

  Consider mass/energy conservation:

  $$\sum_i m_i = \sum_f m_f + Q$$ *initial = final + Q-value*

  So, for above $d + t \rightarrow {}^4He + n$:

  $$\sum_i m_i = m_d + m_t = 2.014102 + 3.016049 = 5.030151u$$

  $$\sum_f m_f = m_{{}^4He} + m_n = 4.002603 + 1.008665 = 5.011268u$$

  $$\Rightarrow Q = \sum_i m_i - \sum_f m_f = 0.01883u = +17.6MeV$$

  +ve *Q* so: Exothermic process

  Energy liberated

- If *A>60*, nucleus splits into 2 fragments, with higher BE than original… therefore energy liberated. (going *up* curve from right)

  Fission.

  e.g. $^{236}U \rightarrow {}^{93}Rb + {}^{141}Cs + 2n$

  Get a *Q* value again, using same formula as above (mass excesses just add/subtract)

  Q = 173.4MeV

  +ve *Q* so: Exothermic…. Energy liberated.

$$Q = \left(\sum_i m_i - \sum_f m_f \right) c^2$$

Corrections to BE function:

1. Volume Energy

$$\frac{BE}{A} = const \text{ , so:}$$

$$\text{Term} = a_V A$$

Where a_V is a volume term with a value of 8MeV

2. Surface Correction

A nucleon near to surface interacts with fewer nucleons than one within the interior, and is hence less tightly bound

$$\text{Term} = -a_S A^{2/3}$$

3. Coulomb Energy

Coulomb repulsion of protons. Coulomb force long range, so p interacts with all other p's.

Also, depends upon A.

Hence: $$\text{Term} = \frac{a_C Z(Z-1)}{A^{1/3}}$$

4. Quantum Mechanical terms

Real nuclei are quantum mechanical systems…

4.1 Symmetry Term

Stable nuclei prefer $N = Z$.

$$\text{Term} = -\frac{a_{sym}(A-2Z)^2}{A}$$

4.2 Pairing Term

Tendency for nuclei that are coupled pair-wise to be more tightly bound.

4 nuclei with odd N & Z

161 with even N & Z

$$\text{Term} = d$$

Combining all these terms, we have:

$$BE(Z,A) = a_V A - a_S A^{1/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - \frac{a_{sym}(A-2Z)^2}{A} + d$$

Hence, the *semi-empirical mass formula* is:

$$m(Z,A) = Z_{{}^{1}H_{1}} - Nm_n - BE(Z,A)$$

A good choice of parameters:

$a_V = 15.5MeV$

$a_S = 16.8MeV$

$a_c = 0.72MeV$ Gives a good fit to the BE/A curve.

$a_{sym} = 23MeV$

$d = 34MeV$

Radioactivity:

$N(t)$ = #nuclei

Measure activity: $A(t)$ = number of disintegrations per second.

$$A(t) = -\frac{dN(t)}{dt}$$

$$A(t) = Be^{-lt}$$ (*A* decays with time…)

Thus: $-\frac{dN}{dt} = Be^{-lt}$ hence $A(t) = lN_0e^{-lt}$, where N_0 is the #nuclei at $t=0$.

Also, $N(t) = N_0e^{-lt}$

l is the decay constant – decay probability per unit time, per nucleus.

Half life: $t_{\frac{1}{2}}$ is the time taken for half the nucleus to decay.

$$t_{\frac{1}{2}} = \frac{\ln\left(\frac{1}{2}\right)}{l}$$

The sum of all lifetimes $\int_0^\infty A(t)tdt$, and substituting in an expression for $A(t)$:

$$lN_0\int_0^\infty te^{-lt}dt = \frac{N_0}{l}$$

Which implies that the average/mean lifetime is given by: $t = {}^1\!/_l$

1 Bequerel = 1Bq = 1 disintegration per sec

1 Curie = 1Ci = 3.7×10^{10} disintegrations per sec

e.g: if $A(t) = 10^4$ decays/sec $N_0 = 10^{23}$ nuclei

then, $l = \frac{10^4}{10^{23}} = 10^{-19}s^{-1}$

In a decay chain, as time increases, the activity of daughter ~ activity of parent: **Secular Equilibrium**.

Alpha decay:

e.g. $^{232}Th \rightarrow {}^{228}_{88}Ra + a$ with $t = 1.4\times10^{10}\,yrs$ and $E_a = 4MeV$

The alpha-emission reduces the nuclear charge, and mass, and hence liberates energy. This also implies that the products are more tightly bound that the initial. The Coulomb barrier of the final nucleus is greater than the KE of the alpha. The only way the alpha escapes is via *quantum tunnelling*.

Beta decay:

A *n* changes to a *p* (or vice-versa) – and the beta particles are electrons that did not 'pre-exist'. A third body is needed – neutrino.

(a) b^- decay $n \rightarrow p + b^- + \overline{u}$ (can be a free *n*, or in nucleus)

$$Q = \{m_n - m_p - m_e - m_{\overline{u}}\}c^2 = 0.782 - m_{\overline{u}}c^2$$

$${}^A_ZX_N \rightarrow {}^A_{Z+1}Y_{N-1} + b^- + \overline{u}$$

(b) b^+ decay $\quad p \to n + b^+ + u$ $\quad$ (p must be within the nucleus)

$${}^{A}_{Z}X_{N} \to {}^{A}_{Z-1}Y_{N+1} + b^+ + u$$

(c) Electron capture $\quad e^- + p \to n + u$ $\quad$ (an atomic electron from an inner shell. Swallowed by atom)

Gamma decay come from nucleons changing from one shell to another.

<u>Interaction of radiation with matter:</u>

$E = cp$ $\quad$ for a neutrino

$E = cp = hu = \frac{hc}{l} \Rightarrow l = \frac{hc}{E} = \frac{h}{p}$ $\quad$ for a photon/neutrino

For alpha-radiation: Trajectory not deflected until it has little energy left. Not very penetrating… short range

Beta particles: Major deflection. Lots of energy transfer. Large deceleration – Bremsstrahlung radiation. Damage spread over wider area… less damaging than alpha.

Gamma rays: Uncharged… weaker interactions… more penetration.

Units & dosage:

Absorbed dose: 1Gray = 1J Kg^{-1}

Dose equivalent: dose in Grays x Q

Radiation type	Q
Gamma-rays all energies	1
Electrons all energies	1
Alpha's, fission fragments	20

Example:

Want: Dose due to naturally occurring ^{14}C in body

${}^{14}_{6}C \to {}^{14}_{7}N$ $\quad t_{\frac{1}{2}} = 5730\,yrs$

Fraction of ^{14}C in all living things = 1.3×10^{-12}

18% of normal body weight (70kg) is carbon

Hence, #atoms of ^{14}C = $\frac{70\times10^3}{12}\times6.023\times10^{23}\times\frac{18}{110}\times\left(1.3\times10^{-12}\right) = 8.2\times10^{14}\,atoms$

$$l = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{5730} = 1.21\times10^{-4}\,yr^{-1} = 3.8\times10^{-12}\,s^{-1}$$

Hence, total activity: $lN = 9.9\times10^{10}\,yr^{-1}$ or 3116Bq

$E_{b_{\max}} = 0.156\,MeV$, so estimate average $E_b = \frac{1}{3}E_{b\,\max}$

So, energy absorbed in 1yr

$$= \tfrac{1}{3}0.156\times9.9\times10^{10}\,MeVyr^{-1}$$

$$= 5.1\times10^{9}\,MeV/yr$$

Therefore, energy per year per kg $= \frac{5.1\times10^{9}\times1.9\times10^{-13}}{17}$

Hence: Absorbed dose = $12mGyr^{-1}$ & Dose equivalent = $12mSyr^{-1}$ (as Q = 1)

Nuclear fission:

Heavy nucleus splits into lighter nuclei.

Lighter nuclei have higher BE… energy released as KE of fragments.

E.g.

$^{239}_{94}Pu \rightarrow ^{112}_{46}Pd + ^{124}_{48}Cd + 3n$

Get a Q value as before = *188.04MeV*.

Need to get over Coulomb barrier…

Estimate 'height' of barrier by imagining bringing together two fragments *Pd* and *Cd*. Assume nucelie uniformly charged spheres, and 'just' touch:

$$E_{coulomb} = \frac{e^2}{4\pi e_0}\frac{Z_1 Z_2}{r}$$

Use fine structure constant: $\alpha = \frac{e^2}{4\pi e_0 \hbar c} = \frac{1}{137} \Rightarrow \alpha \hbar c = \frac{e^2}{4\pi e_0} = \frac{197}{137} MeV.fm$

$r = r_0\left(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}}\right) = r_0\left(112^{\frac{1}{3}} + 124^{\frac{1}{3}}\right) = 11.8\,fm$ Since $r = r_0 A^{\frac{1}{3}}$ (where $r_0 = 1.2fm$)

So, $E_{coulomb} = \frac{197}{137} MeV.fm \times \frac{48 \times 46}{11.8\,fm} = 270\,MeV$

Only 188MeV from fission energy, and a 270MeV Coulomb barrier.

So, *classically*, fission cannot occur; but *quantum mechanically* it can, via QM tunnelling. Like in alpha-decay.

This process is **spontaneous fission**. But has a very long half-life, and is then very unlikely.

Another way to induce fission is by exciting the nucleus, so it goes above Coulomb barrier. Do this by absorbing a neutron.

e.g. $^{235}U + n \rightarrow ^{236}U^*$ U^* means in an excited state.

$kT = \frac{1}{40} eV$

But still below the barrier – so need to supply additional KE via the *n*.

… Enrichment.

Fragments…

Fragments in fission processes extremely rich in neutrons.

- get rapid shedding of neutrons: $n + ^{235}U \rightarrow \eta = 2.4$ and $n + ^{239}U \rightarrow \eta = 2.8$

η is the average number of neutrons produced.

Initial fragments are highly radioactive – decay to stable nuclides.

Each *n* can initiate another fission process – chain reaction.

Delayed neutrons:

The *n*'s here: $^{235}_{92}U_{143} \rightarrow ^{93}_{37}Rb_{56} + ^{141}_{55}Cs_{86} + 2n$ are **prompt neutrons**.

Emitted at instant of fission ~10^{-22}secs

But the *n* in the subsequent reaction $^{93}_{37}Rb \xrightarrow{\beta-decay} ^{93}_{38}Sr_{55} \rightarrow ^{93}_{38}Sr_{54} + n$ is a **delayed neutron**. They are delayed by the $t_{1/2}$ of *Rb* ~ 6secs.

This delay allows mechanical control of reaction rate.

Reaction rate depends upon:

Ingredients

Flux of incident particles

Number of target atoms $= N_p dxA$

Physics of process… rate $= N_p dxAf s$

s is the cross-section = relative probability of process; or thermal capture cross-section.

Fission rate $= Nsf$ = #atoms x thermal capture cross-section x flux = fissions/sec

In addition to fission, other processes occur from ${}^{235}U + n \rightarrow {}^{236}U^*$, so can have

${}^{236}U^* \rightarrow {}^{235}U + n$	elastic scattering
${}^{236}U^* \rightarrow {}^{235}U^* + n$	inelastic scattering
${}^{236}U^* \rightarrow {}^{236}U + g$	radiative capture

Each process has a probability, given by corresponding cross-section s_i.

Also, $s_{scattering} = s_{elastic} + s_{inelastic}$ and $s_{absorption} = s_{fission} + s_{capture}$

Need to slow neutrons down to maintain reaction – **moderation**.

Mean free path for neutrons: $l_s = \dfrac{1}{Ns_s}$

Chain reactions:

Each fission produces an average h neutrons – these will lead to other fission reactions. - a chain reaction.

But some neutrons are lost (e.g. through radiative capture) and to do not contribute towards chain reaction.

Neutron reproduction factor k to account for these losses.

k gives net change in #*n* from one generation to next.

k = 1	critical	#*n* constant	want for stable energy production
k > 1	supercritical	#*n* rises	bomb!
k < 1	subcritical	#*n* falls	nothing happens

Criticality – evolution of #*n* with time…

Assume *N(t)* neutrons at *t = 0*

#absorbed in next *dt* = $N(t)\dfrac{dt}{t_a}$ t_a = mean time before absorption

Each of these neutrons release k useful neutrons per absorption:

i.e: $$dN = N(t)k\frac{dt}{t_a} - N(t)\frac{dt}{t_a}$$

So: $$N(t) = N_0 e^{\frac{(k-1)}{t_a}t}$$

Energy released in fission:

Rate of change of energy = *Q*-value per absorption x #absorbed in time dt

$$dE = Q.\frac{N(t)}{\tau_a}dt = QN_0 e^{\frac{k-1}{\tau_a}}\frac{dt}{\tau_a}$$

Integrating gives:

$$E(t) = \frac{QN_0\tau_a}{(k-1)\tau_a}e^{\frac{k-1}{\tau_a}t}$$

<u>Nuclear fission reactors:</u>

Require chain reaction under carefully controlled conditions.

k ~ 1

Designed to be less than critical for prompt neutrons, but just above for delayed neutrons.

About 80% of the energy is released as KE of fission fragments – loose this to surroundings, which is used to produce steam to turn turbines.

Enrichment:

Cannot use chemical reactions to separate one isotope from another.

Naturally occurring U is 99.3% U^{238}
0.7% U^{235}

U^{235} is the only naturally occurring material with reasonably large σ_f - fission cross-section.

Methods:

1) Electromagnetic process: – mass spectrometer idea.

Forces balance when $Eq = qvB \Rightarrow v = \frac{E}{B}$

Then, $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

Hence, different masses fall at different radii

2) Gas diffusion:

Uses different diffusion rates of UF_6 gas through a porous material

Very expensive, due to pumping costs.

3) Gas centrifuge:

Put gas into rotating centrifuge, and $^{235}UF_6$ is lighter molecule, so will collect near axis of rotation.

Many cycles required, but uses 1/10 power of diffusion method!

4) Laser ionisation:

i) Produce atomic beam of U
ii) Use a 1st, narrowband laser to excite only ^{235}U, giving $^{235}U^*$
iii) Use 2nd, broadband, laser to only ionise $^{235}U^*$
iv) Separate ionised $^{235}U^+$ from neutral ^{238}U using electrostatic deflector.

Under development, but should be a lot cheaper

Moderation:

Neutrons of few MeV, means low fission cross-section

So, slow them down to give higher σ_f.

Use conservation of momentum, and KE (*v* is initial velocity of *n*, and u_i the final velocities), to do calculations…

$$\frac{\frac{1}{2}mu_1^2}{\frac{1}{2}mv^2} = \left(\frac{M-m}{M+m}\right)^2$$

, so a *n* scattered from a heavy nucleus, gives hardly any energy-loss at all.
So, to reduce *n*'s energy from 1MeV to 1/40eV, using Carbon (as its cheap), take many collisions. After N collisions:

$$\frac{E_{final}}{E_{initial}} = \frac{1/40}{10^6} = \left(\frac{11}{13}\right)^{2N}$$

Giving $N \sim 50$.
This was over optimum scattering angle (180deg), so an average over all, would need ~100 collisions.
Using D_2O (deuterium – heavy water), which has virtually no neutron capture, is best.
A good moderator material should be:

- light,
- large s_s (scattering cross-section),
- small s_a (absorption cross-section)

Reactor control:
Require a control system to be able to bring about small changes in k, from one generation to the next.

Start-up (increase k), during operation, shut down (decrease k).

This is done using control rods. Materials which have high absorption cross-section, s_a, for neutrons.

Good materials are Boron (B) and Cadmium (Cd).

The existence of delayed neutrons allows this control mechanism to be achieved.

Types of fission reactors:

1) Boiling water reactor:

Problem is that water circulating through core becomes radioactive.

2) Pressurised water reactor:

Water is pressurised, enough so that it cannot turn into steam, and passed through the reactor. This water heats a second water system within a heat exchange, which then delivers steam to a turbine. This second water is not radioactive.
Used in UK with air instead of water, and graphite as the moderator. Also used liquid sodium instead of circulating water – as remains liquid at higher temperatures.

Typical operating conditions:

Fission rate = #atoms x fission-cross-section x thermal flux

$= N\sigma_f \phi$

So, for 100tonnes U, and 10^{13}n/cm^2/sec thermal neutron flux $(= \phi)$:

$$\#\text{U atoms} = \frac{100\times10^6}{238.05}.6.02\times10^{23} = 2.5\times10^{29}$$

0.72% of which are $^{235}\text{U} = 1.8\times10^{27}$

Giving a fission rate of $1\times10^{19}s^{-1}$

Remember that about ***200MeV/fission*** is given off, so for our reaction,

$2\times10^{21}MeVs^{-1} = 400MW$ is emitted.

So, mass used = #fissions per sec x mass of ^{235}U atom

$$= 1\times10^{19}\frac{235g}{N_A} = 4mg/s = 346g/day$$

As fuel is used up, neutron flux falls, so need to move control rods out to keep reaction stable.

For the shape of the core: need to minimise the number of n's leaving, so have spherical shape for the core…. Minimise surface/volume ration.

Efficiency:

Of an ideal engine: $= \dfrac{T_{core} - T_{condensor}}{T_{core}}$

Using T_{core} = 600K $T_{condensor}$ = 300K

Gives a maximum efficiency of 50%. Which wont be achieved, more like 30% will be.

So need a higher T_{core}. Use sodium!

Nuclear fusion:

Fuse together two light nuclei, to get a more tightly bound compound system – energy released.

e.g: $d + t \rightarrow \alpha + n$ with a Q value of 17.6MeV
deuterium and tritium
energy released as the KE of the fragments.

How is this energy shared between α & n?

Use conservation of momentum to get: $m_\alpha v_\alpha = m_n v_n \Rightarrow v_\alpha^2 = \frac{m_n^2}{m_\alpha^2} v_n^2$

And, conservation of energy: $\frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_n v_n^2 = Q$

And, a ratio of KE's gives: $\frac{\frac{1}{2} m_\alpha v_\alpha^2}{\frac{1}{2} m_n v_n^2} = \frac{\frac{1}{2} m_\alpha}{\frac{1}{2} m_n v_n^2} . \frac{m_n^2 v_n^2}{m_\alpha^2} = \frac{m_n}{m_\alpha} = \frac{1}{4}$

So, the *neutral* neutrons end up with most of the energy.
But their being neutral makes it hard to capture their energy within the plasma.
So, plasma cools down.

To initiate fussion reaction, need to bring $d + t$ close together – above the Coulomb barrier.

$$\text{Coulomb barrier} = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2}{(r_d + r_t)} = \frac{197}{137} \frac{1 \times 1}{\left(2^{1/3} + 3^{1/3}\right)\underbrace{1.2}_{r_0}} = 0.44 MeV$$

Get this energy in a number of ways:
1) Particle accelerator… beam power needed (calculate from #d's per sec x Q) = 17.6W, which is not very economical for power production.

2) Heat up d+ t gas Plasma! (a neutral gas of ions and e^-'s)
Needs thermal energies > Coulomb barrier!
kT = 0.44MeV thermal energy
Giving a T of $5 \times 10^9 K$!!

Two other effects help however: QM tunnelling through barrier; d + t gas has a distribution of speeds, and so energies.
Which, combined, allows fusion at $10^6/10^7$K.

Other possible fusion reactions:

$d + d \rightarrow {}^3He + n \quad Q = 3.3 MeV$

$d + d \rightarrow t + p \quad Q = 4 MeV$

$d + t \rightarrow \alpha + n \quad Q = 17.6 MeV$

All have same Coulomb barrier, but the one with highest Q is the most likely.

So, how can thermal fusion be obtained?
1) TOKAMAK – like a doughnut shape. Confines hot plasma magnetically, and currently under development.
2) Laser induced fusion – compress & heat *d/t* plasma using a laser
Can also use process of muon-catalysis – essentially 'cold fusion' – never reproduced though!

Fusion reaction properties:

For the reaction $d + t \rightarrow a + n$

Reaction rate $= Ns_f f$ $\qquad s_f$ is like the probability of reaction!

Assume: 1) Plasma of tritium & deuterium, with densities n_t and n_d per m^3

2) Relative velocity of d,t nuclei = v ms^{-1}

So, can simplify, by saying all t fixed, and d moves at velocity v.

i.e. have a flux of deuterium = $n_d v$

Hence, fusion rate $= n_t s_f n_d v$ per unit volume.

Need to take an average for v, as a Maxwell-Boltzmann dist.

So, fusion rate $= n_t n_d \langle s_f v \rangle$.

Plasma has a thermal energy: $\sum \frac{3}{2} kT = \frac{3}{2} kT (n_t + n_d + n_e)$

Assume a neutral plasma, so $n_t = n_d = \frac{1}{2} n$ & $n_e = n$ (twice as many electrons…)

So, $\frac{3}{2} kT (\frac{1}{2} n + \frac{1}{2} n + n) = 3kTn$.

This is the energy needed, to keep plasma at temp T.

The energy out:

Say plasma produced for a time length t, each reaction producing Q-energy.

$$\text{Therefore, energy released} = \underbrace{n_d n_t \langle sv \rangle}_{rate/\sec} . \underbrace{Q}_{energy/reaction} . \underbrace{t}_{confinement\_time}$$

$$= \frac{1}{4} n^2 \langle sv \rangle Q t$$

So, if we want to *produce* energy:

Fusion energy > thermal energy supplied.

i.e:

$$\frac{1}{4} n^2 \langle sv \rangle Q t > 3kTn$$

Or:

$$nt > \frac{12kT}{Q\langle sv \rangle} \qquad \text{Lawson Criteria}$$

$\langle sv \rangle \sim 10^{20} sm^{-3}$

BREAKEVEN: energy out = energy in

Basically, higher temperature will require less confinement time

Self-sustaining fusion reactions:

BREAKDOWN: Power *produced* by fusion reaction = power *needed* to heat plasma.

Currently approaching this.

IGNITION: Power *produced* by fusion reaction can *maintain* reactor, with *no* external source of energy

Not yet been achieved.

For the $d+t \rightarrow \alpha + n$ reaction, with a Q of 17.6MeV, 3.52MeV goes to the α, and the other 14.1MeV goes to the n.
Now, α-particles are charged, so are confined by the B-field, so transfer their energy to the plasma to keep it hot.
The n carries most of the energy, as lighter; are neutral, so escape plasma. They are captured in the walls of the reactor, where they generate heat – steam – turbine – generator.

n's are also used to breed further tritium, via ${}^{6}Li + n \rightarrow t + d$, where the ^{6}Li is contained in a blanket around the reactor.

Fusion devices:

1) Magnetic confinement:

Charged particles (i.e. the plasma) spirals in a uniform B-field.
Make B-field go round in a big circle, so plasma spirals in a circle.
This is called a TOKAMAK (Russian acronym)
e.g. JET @ Oxford

To get B-field like this, have two contributions:
One along toroidal axis – one around axis.
And the combination gives a helical B-field along the toroidal axis, which confines the charged particles.
The B-field along the axis is produced by 32 field coils.
The B-field around axis produced by inducing a current in the plasma using a transformer – plasma acts as the secondary coil to the transformer.

To get the plasma hot enough, use:

a) Radio frequency heating – RF waves 'drive' electrons in plasma.
b) Neutral beam heating – inject 10-100keV beam of H or D into plasma – beam collides with plasma, and gives energy off.

2) Inertial confinement:

The fuel (d/t & plasma) is compressed to very high densities, for very short confinement time (Lawson Criteria…).
Do this by:

a) Small pellet of fuel ~0.1mm diameter, contains d/t mixture;
b) Strike by laser beams from all directions;
c) Laser beam heats surface, producing shock waves which implode the pellet;
d) Giving a high T and density;
e) Giving a fusion reaction.
Typical pulse ~10^{5}J in 10^{-9}s = 10^{14}W instantaneous power.

- Presently under development.

<u>Renewable energy sources:</u>

Current dependence on oil is dangerous:

Volatile political situation, and pollution are problems.

Solutions:

Nuclear energy:	Fission – supplies ~30% UK's electricity, but will be decommissioned in ~2035.
	Fusion – about 30yrs away
Renewable energy sources	
	Wind/solar/tidal – government wants ~10% by 2010, 20% by 2020. Currently 3%

Greenhouse gases:

CO_2 have molecular vibrations, and strongly absorb radiation in far-IR.

Its vibrational frequencies $\sim 3\times10^{13} s^{-1}$, so, as $c = l n \Rightarrow l = \frac{3\times10^{8}}{3\times10^{13}} = 10 mm$ - far IR.

Surface temp of sun ~6000K, and suns radiation peaks at ~500nm (black-body radiation…).

Earth at a much lower temperature (~300K), and its radiation peaks at $\sim 10 mm$.

Earths atmosphere is strongly transparent to visible wavelengths, and also near IR. So suns radiation passes straight through.

However, CO_2 molecules *strongly absorb* the earth's far-IR radiation, and so acts like a blanket, leading to an increase in the earth's surface temp.

Note though, if no greenhouse effect, then earths equilibrium temp would be ~35K cooler.

Define a renewable energy source:

> ***Where we extract energy from continuing, or repetitive energy currents, which are naturally occurring in the environment.***

e.g. sun, tides, wind.

Also, non-renewable energy sources, and ones where we extract energy from finite sources of energy (e.g. fossil fuels…).

Global figures:	Sun:	120,000TW (1TW = 10^{12}W) absorbed on earths surface
	Wind:	1200TW
	Tidal:	3TW

But, local sites can be hard, due to environmental conditions.

e.g. windy, but not much sun in UK.

So, look for 'local solutions'.

<u>Wind power:</u>

- An established technology.

Extraction…

In 1sec, air molecules move distance u_0.

Mass of air passing point per sec $= rAu_0$ A = area of blade

Flow of KE past point per sec = $P_0 = \frac{1}{2}(mass - flow)(u_0)^2$

So:

$$P_0 = \tfrac{1}{2} rAu_0^3$$

- The max power available.

Note dependence on wind speed u_0!

Unfortunately, cannot extract all this power – otherwise all the molecules would 'bunch up' with zero velocity and build up in the turbine.

Betz criterion: ***Only ½ wind power can be extracted***

But practical, commercial uses ~40% available power.

So, siting of turbines very important.

Wind speed increases dramatically with height, but density drops.

So, best position: a dome shaped hill, clear of other turbines.

Advantages/disadvantages:

- Eyesore – to some!
- High cost ~2x nuclear/conventional power stations
- Direction & wind speed variable, described by "load factor" ~30% (i.e. 70% of time wind too slow/in wrong direction…)
- Need a lot of turbines in a farm to replace convential/nuclear plants ~200
- Power per m^2 is low – need to be separated by ~5/10x blade length to stop interference
- No greenhouse gases given off
- Non polluting once built
- Renewable energy source

<u>Solar power:</u>

Comes from nuclear fusion in the sun:

Proton-proton cycle:

$$p + p \rightarrow d + e^+ + n$$
$$p + d \rightarrow {}^3He + g$$

these are repeated to give: ${}^3He + {}^3He \rightarrow a + 2p$

So, the net process is: $4p \rightarrow a + 2e^+ + 2n + 2g$

e^+ annihilate releasing energy as gamma-rays.
n escape as weakly interacting.

Total energy released = 20.02MeV, or 6.55MeV/p

So, how long will the sun survive…?

Total solar luminosity $= 4\times10^{26}W = \frac{4\times10^{26}}{1.6\times10^{-19}\times10^{6}} MeV/s$
So, #p's consumed per second $= 4\times10^{38}s^{-1}$
Solar mass $= 2\times10^{30}kg$, so sun can last for 10^{10} years.

Power density at earths radius ~1.4kWm^{-2}

- it's a fairly constant source.
  Fluctuations in suns output $\pm1.5\%$
  Fluctuations due to sun-earth distance $\pm$ 4%

How do we harvest this solar energy…?

Note, biofuels/food – photosynthesis – has 6% efficiency

1) Direct heating:

Net energy gain:

$$= GA - \underbrace{k\left(T_{fluid} - T_{surroundings}\right)}_{conduction} - \underbrace{s\left(\Delta T\right)^4}_{\substack{radiation \\ Stefan's-law}}$$

Points:

a) thermal insulation;
b) shelter from wind minimise convection losses
c) select a surface to absorb at peak wavelengths;
d) solar concentrators – parabolic mirrors.

Suggests "local solutions":
e.g. roof-top water heaters in Cyprus.

This is "low grade" heat – low efficiency (Carnot cycle, temp of surroundings)

2) Photocells
These use a pn junction to convert photons into an electric current.

A pn junction is made by joining together pieces of p-type and n-type semiconductor.

If the photon energy is greater than the band gap

i.e. $hn > E_{gap}$

then the photon promotes an electron from the valence band to the conduction band, leaving behind a +ve ‘hole’.

The e^- rolls down the hill, due to electric potential. And the e^+ rolls up the hill.

Producing an electric current;

Which is delivered to an external circuit.

- The *Photovoltaic Effect*.

Photocell current ~ 200Am^{-2} in full sun.

Photocell voltage ~ 0.5V

So, power = IV = ~100Wm^{-2} (about enough for an electric light bulb)

Efficiency ~ 10-15%

Losses due to:

a) Photon energy – if $hn < E_{gap}$ - about 25% loss

e.g. $E_{gap}(Si) \sim 1.1eV \Rightarrow l \sim 1.1mm$

b) Photon heating – excess energy heats up the crystal.

To get higher voltages, connect cells in series

- which is what a solar panel is.

Advantages/disadvantages:

- renewable energy source
- no pollution
- free energy once built
- expensive – very high purity Si material, which is expensive to cut into wafers
- need a sunny location
- need lots of them
- low efficiency

- New materials are being tested.