
Experimental Tests of Special Relativity

57976200

The University of Manchester

Introduction to Special Relativity:^{1,2}

In 1905, Albert Einstein published a paper on the theory of Special Relativity. The theory relied upon two postulates:

1. Any law of physics which holds in one coordinate system also holds in identical form in any other coordinate system moving uniformly with respect to the first (i.e. is an “inertial frame”)¹;
2. In any inertial frame, the speed of propagation of light in vacuo is the same in all directions².

This theory carried with it a number of consequences, some (if not all!) of which are counter-intuitive. They include ideas about time dilating, and ones about space contracting, when you travel at a large fraction of the speed of light. The speed of light is usually denoted by c , and is equal to $c = 299\,792\,458\text{m/s}$, which is usually approximated to $3 \times 10^8\text{m/s}$.

A brief introduction of these ideas follows.

Postulate 1:^{3,4}

This states (as an example), that Maxwell’s equations for electricity and magnetism³:

$$\begin{aligned} \nabla \cdot \underline{E} &= \frac{\underline{r}}{\underline{e}_0} & \nabla \cdot \underline{B} &= 0 \\ \nabla \times \underline{E} &= -\frac{\partial \underline{B}}{\partial t} & \nabla \times \underline{B} &= \underline{m}_0 \underline{j} + \underline{m}_0 \underline{e}_0 \frac{\partial \underline{E}}{\partial t} \end{aligned} \quad (1)$$

work in an identical form in different inertial frames, in vacuo. That is, to go from one frame to another, all one must do is to use ‘primes’ on one set of equations. Infact, the speed of light can be derived from these equations; after using various identities, and noticing that the wave equation is produced⁴:

$$\nabla^2 \underline{E} = \frac{1}{c^2} \frac{\partial^2 \underline{E}}{\partial t^2} \quad (2)$$

With:

$$c = \frac{1}{\sqrt{\underline{e}_0 \underline{m}_0}} \quad (3)$$

And one will notice that c depends only on the values of two constants - \underline{e}_0 (the permittivity of free-space), and \underline{m}_0 (the permeability of free-space). The process of swapping from one frame to another does not affect the value of c .

Hence Postulate 2 can be derived from Postulate 1.

Postulate 2:^{1,5,6,7}

Suppose you were on the surface of the earth. And you travel at 10m/s in a car, along a road. That is 10m/s *relative* to the “stationary” road. Now, suppose you shine a torch forwards, away from the car, in the direction of motion. Now, you, in the car (that is, a person moving at $v = 10\text{m/s}$) see the light recede at a speed of $c = 3 \times 10^8\text{m/s}$.

Now, suppose that there is a person on the side of the road, stationary relative to the road, who sees all this going on.

One may initially think, relative to the road, the “observer”, may see the light to be travelling at $c' = c + v = 3 \times 10^8 + 10\text{m/s}$. However, Postulate 2 says otherwise! Postulate 2 says that the speed of light is always the same, no-matter of how fast the “source” is moving. So infact, the observer will only see the light to be travelling at $c' = 3 \times 10^8\text{m/s}$.

To think about this more generally, think of the following construction. See Fig 1. for a representation of the two frames.

Suppose you have a source of light which is moving at speed v relative to a stationary observer. The frame in which the source is stationary is labelled Σ , and the frame in which the observer is stationary is S . Now, light is emitted from Σ towards S , at speed

c. So light is emitted backwards from the direction of motion.

You may initially think that the observer will see the light to be travelling at speed $c' = c - v$. However, Postulate 2 says that $c' = c$.

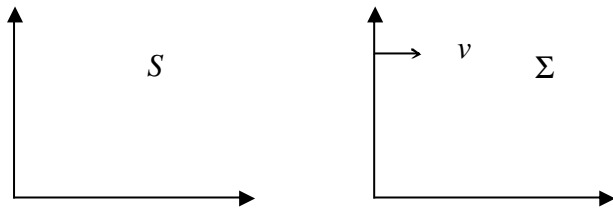


Fig 1. Frames of Reference

Another interesting implication of Postulate 2 is that of “length contraction”, or Fitzgerald Contraction. This is all tied in with the ideas above. However, a fair deal of in-depth maths is involved with obtaining the below equations⁵.

Suppose (using the same notation for S and Σ above, moving at speed v) you measure a rod which is stationary in Σ , and you find it has a length L_0 . If an observer in S now does the same thing, to the rod in Σ (which, you remember is moving at speed v). When Postulate 2 is taken into account, and the maths done, one finds that the length of a moving rod, as measured from a “stationary” observer is *less* than the length measured by the person moving *with* the rod. The amount by which these two values differ can be calculated, and is given by:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{g} \quad (4)$$

Where:

$$g = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5)$$

You may notice, in (4), if v is a lot less than c ($v \ll c$), then $L \approx L_0$, as $g \approx 1$. Which is what you would expect.

However, as an example, if $v = 2.7 \times 10^8$ m/s (90% of the speed of light), then the “gamma factor” in (5) is $g = 2.294$. So, for a rod which is measured to be 10m long in Σ , is found to be 4.359m long in S ! It has shrunk!

One may notice that you have to be very careful over how you express speeds, and relative to what. And about what one can say to be stationary. A rod is measured in “its” stationary frame, but the frame

itself is moving *with respect* to another, which may then be moving with respect to another.

This is another by-consequence of special relativity – you cannot absolutely define “at rest”.⁶

The idea of “time dilation” is even odder!

Again, it rests upon Postulate 2, and requires a lot of maths to prove⁷, but the basic idea is outlined below:

Suppose you have two clocks, which are identical, and tell exactly the same time when brought together. One clock stays on the earth, and the other is placed on a space-ship. The space-ship flies away from the earth, at a speed v (being careful here about the “constant velocity” concept) attaining an incredible fraction of c . And then comes back to earth.

The two clocks are then brought together again. The clock which went on its trip to the stars reads a different time to the one which stayed on the earth. Infact, it is showing a time *before* the other one. “Moving clocks run slow”.

Suppose the amount of time which has passed on the “moving” clock is t_0 , and the amount of time on the “stationary” clock (the one on the earth) is t . Then, the discrepancy between the two times is given by:

$$t = t_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = t_0 g \quad (6)$$

Again, a numerical example:

If a clock moving at 90% of c ($v = 0.9c$) registers a time period of a day ($t_0 = 24$ hrs), then as $g = 2.294$ (as before), the amount of time passed by a clock which was stationary to it is $t = 55$ hrs! This means that more time has passed on earth than on the space-ship. A day passed on the space-ship, but a little over two days passed on earth!

These bizarre *predictions* by Special Relativity do actually have experimental back-up. It is an outline of a selection of these experiments that will take up the remainder of this paper.

Experimental Tests of Special Relativity:^{1,8,9,10,11,12}

One of the most famous results of special relativity is that of the equivalence of mass and energy:

$$E = mc^2 \quad (7)$$

This can be derived from relativistic equations for work⁸.

The concept of energy being proportional to mass is best demonstrated in a nuclear reaction.

For example, when deuterium d and tritium t nuclei undergo fusion in the following reaction (where n is a neutron):



Now, the masses of the ingredients and the mass of the product are known⁹. If one finds the difference between the sums of the masses of the products, and those of the ingredients:

$$\Delta m = (m_d + m_t) - (m_a + m_n) \quad (9)$$

One finds that $\Delta m \neq 0$. If the Δm value is converted into MeV , via (7), one will find that $\Delta m = 17.6 MeV/c^2$. This is known as the systems “binding energy”, and is released when reaction (8) takes place.

This predicted value has been verified experimentally. At Oxford’s JET fusion reactor¹⁰, for example.

A paper published in 1977, by K.Breecher¹, outlined results from an experiment which used a “Doppler” type effect which would be observed from distant binary systems. The effect would be visible if special relativity was incorrect.

Infact, Breecher starts his paper stating another theory – Ritz’s idea, called “emission theory” from 1908. This theory says that electromagnetic radiation (light, travelling at speed c), emitted from a moving source (source moving at a speed v), would be perceived to be travelling at a speed $c' = c + v$ relative to a stationary observer. This is actually what one would intuitively expect. You can then go on to say:

$$c' = c + kv \quad (10)$$

You will notice that $k = 0$ for special relativity, and $k = 1$ for emission theory.

It will then be possible to derive an experiment which could measure this ‘ k ’ value.

W.de.Sitter in 1916 did just this¹¹. He proposed that if the velocity of light did indeed depend upon the

velocity of the source, then binary star systems *should* show “peculiarities”. These effects included apparent “extra” eccentricity of orbits, and multiple images of stars. The absence of such effects had been taken as evidence against emission theory.

However, J.Fox¹² in 1965 suggested that the interaction of light with an interstellar medium, would counter this effect. This interaction would essentially mean that after a certain distance (its extinction length), light would disperse. And would simply not reach earth. However, it was realised that if a certain wavelength was measured, x-rays infact, then these dispersion effects would not be able to invalidate any perceived peculiar effects.

Infact, Fox details an experiment which measured the ‘ k ’ value, without any extinction processes – one using the decay of p^0 mesons. This is due to the decay products of p^0 mesons: they decay into g -rays. Light. This is then effectively a moving source of photons, from which measurements can be taken. He gives the result as $k = 0$, with an accuracy of 0.0001.

Despite this, Fox¹² concluded his paper stating that although there has been experimental observations to back up special relativity, the same experiments did not actually null the Ritz theory.

After some mathematics, one can insert data from known properties of certain star-systems, into equations, to find a limit to the ‘ k ’ value. Breecher found this to be:

$$k < 2 \times 10^{-9} \quad (11)$$

The data used by Breecher, to get to this value of k , came from the Her X-1 binary x-ray system¹. Other, smaller, values of k were found from other systems, but they were not as reliable as the Her X-1 source.

It may trouble you that this value is not zero. However, due to the nature of experimental physics, it would be almost impossible to get a 100% accurate value.

This value of k was small enough for Breecher to conclude that Postulate 2 is correct:

“...require that the velocity of electromagnetic radiation (x rays) be independent of the motion of the source to a high degree of accuracy.”¹

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