

Coulomb's Law:

$$\underline{F} = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \hat{r}$$

Electric Potential:

From: $W = q \int_A^B \underline{E} \cdot d\underline{l}$

Potential difference:

$$f(r_a) - f(r_b) = - \int_a^b \underline{E} \cdot d\underline{l}$$

Potential in going from infinity to r :

$$f(r) = \frac{q}{4\pi \epsilon_0 r}$$

$$\underline{E} = -\nabla f$$

In electrostatic fields *only*: $\nabla \times \underline{E} = 0$

Thus \underline{E} is conservative.

Electric Dipole:

$$f(x, y, z) = \frac{1}{4\pi \epsilon_0} \left(\frac{q}{\sqrt{x^2 + y^2 + (z-d)^2}} + \frac{-q}{\sqrt{x^2 + y^2 + (z+d)^2}} \right)$$

For the off-axis potential, for two charges $-q$ and $+q$, separated by d .

Gauss' Law:

Integral form:

$$\oint_S \underline{E} \cdot d\underline{S} = \frac{1}{\epsilon_0} Q_{enc} = \frac{1}{\epsilon_0} \int_V \rho dV$$

Differential form:

$$\nabla \cdot \underline{E} = \frac{\rho}{\epsilon_0} \quad - \text{Maxwell's 1st equation}$$

Thus:

$$\nabla^2 f = \frac{\rho}{\epsilon_0} \quad - \text{Poisson's equation}$$

If no charge densities:

$$\nabla^2 f = 0 \quad - \text{Laplace's equation}$$

Method of Images:

Same boundary conditions => same solution

Capacitors & Dielectrics:

$$C = \frac{Q}{V}$$

Find V from $\int_a^b \underline{E} \cdot d\underline{l}$

A dielectric material is an insulator: thus conductivity is zero.

Dielectrics gain dipole moments when placed in an electric field.
i.e. they polarise

$$C = \epsilon C_0$$

As dielectric inserted, capacitance increases.
Thus reduces the internal electric field.

Mechanisms for polarisation:

Electronic:

The atoms within the dielectric shift, their electron clouds moving, hence leaving the atom with a dipole... which then align with the applied electric field.

Aligned dipoles:

Works in a similar way, but this time the intrinsic dipoles of the molecules align – when there is no external field present, they are at “random”.

Dipole moment:

$$\underline{p} = q\underline{d}$$

Polarisation: $\underline{P} = n\underline{p}$ like the total number of dipole moments.

Surface charge density:

$$\underline{s}_p = \underline{P} \cdot \hat{n}$$

Where \hat{n} is a normal to the surface.

$$\begin{aligned} \underline{P} &= (\epsilon - 1)\epsilon_0 \underline{E} \\ \epsilon_E &= \epsilon - 1 \\ \underline{r}_p &= -\nabla \cdot \underline{P} \end{aligned}$$

In the function $\underline{P} = (\epsilon - 1)\epsilon_0 \underline{E}$, we assumed:

Linear, isotropic, homogeneous & non-conducting

Electric Displacement Vector:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P}$$

Hence, a new version of Gauss' law:

$$\begin{aligned} \nabla \cdot \underline{D} &= \mathbf{r}_f \\ \oint_S \underline{D} \cdot d\underline{S} &= \int_V \mathbf{r}_f dV \end{aligned}$$

\underline{D} is not affected by polarisation charges.

\underline{P} exists only within the dielectric.

The total charge density is the sum of the free charge density with the polarisation charge density:

$$\mathbf{r} = \mathbf{r}_p + \mathbf{r}_f$$

Boundary Conditions:

D_{perp} Continuous across boundaries

E_{\parallel} Continuous across boundaries

$$\begin{aligned} D_1 \cos \mathbf{q}_1 &= D_2 \cos \mathbf{q}_2 \\ E_1 \sin \mathbf{q}_1 &= E_2 \sin \mathbf{q}_2 \end{aligned}$$

The energy density can be given by:

$$U = \int_V \frac{1}{2} \underline{D} \cdot \underline{E} dt$$

Also, Poisson's equation becomes:

$$\nabla^2 \mathbf{f} = - \frac{\mathbf{r}_f + \mathbf{r}_p}{\epsilon_0}$$