

Magnetostatics:

Lorentz force law:

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

Motion of a charge in magnetic field, with everything at right angles:

$$F = \frac{mv^2}{r} = qvB$$

Period:

$$t = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Thus, cyclotron frequency is:

$$\mathbf{n} = \frac{qB}{2\pi m}$$

Maxwell's 2nd Equation:

$$\nabla \cdot \underline{B} = 0$$

Current:

$$I = Ne v_d A$$

Ohm's Law:

$$\underline{j} = \mathbf{s} \underline{E}$$

Force on a wire:

$$\underline{F} = I d\underline{l} \times \underline{B}$$

$$d\underline{B} = \mu_0 I \frac{d\underline{l} \times \hat{r}}{4\pi r^2}$$

Which is from the Biot-Savart law... gives the \underline{B} -field from a wire carrying a current.

Ampere's Law:

$$\oint_l \underline{B} \cdot d\underline{l} = \mu_0 \sum I$$

Or:

$$\nabla \times \underline{B} = \mu_0 \underline{j}$$

static case only!!

Magnetic Vector Potential:

$$\underline{B} = \nabla \times \underline{A} \qquad \underline{E} = -\frac{\partial \underline{A}}{\partial t} - \nabla \phi$$

And:

$$\underline{A} = \frac{\mu_0 I}{4\pi} \int \frac{d\underline{l}}{r} \qquad \text{a form of the Biot-Savart law}$$

Magnetostatics in Materials:

Inductance, in vacuum, is:

$$L_0 = \mu_0 N^2 \pi r^2 l$$

And in materials is:

$$L = \mu L_0 \qquad \mu = \frac{1}{1 - \epsilon_B}$$

Magnetic dipoles:

Magnetic moment = current X area

$$\underline{m} = I \underline{A}$$

Torque on current loop:

$$\underline{\tau} = \underline{m} \times \underline{B}$$

Potential energy of loop:

$$U(\mathbf{q}) = -\underline{m} \cdot \underline{B}$$

Magnetisation is like polarisation: total dipole moment per unit area:

$$\underline{M} = N \underline{m} \qquad \text{unit: amp.m}^{-1}$$

Thus:

$$\underline{M} = \epsilon_B \frac{\underline{B}}{\mu_0} \qquad \epsilon_B = \text{magnetic susceptibility}$$

Forms of Magnetisation:

Diamagnetism: from individual atoms, and electrons orbiting in \underline{B} -fields
 Paramagnetism: moments aligning
 Ferromagnetism: due to domains within material

Hysterisis: in Ferro-magnets: process by which the material retains its magnetisation after external field removed.

Dia	Weak	Linear	-ve
Para	Stronger	Linear	+ve
Ferro	Strong	Non-linear	Permanent

Magnetic dipoles in materials can be visualised as small current loops, where the internal components cancel out, to leave only a surface current.

$$\underline{i}_s = \underline{M} \times \hat{n} \quad \underline{i}_b = \nabla \times \underline{M} \quad \Rightarrow \underline{i} = \underline{i}_s + \underline{i}_b$$

$$\underline{B} = \underline{B}_0 + \underline{m}_0 \underline{M}$$

Thus, introduce the magnetic field vector:

$$\underline{H} = \frac{\underline{B}}{\underline{m}_0} - \underline{M} \quad \Rightarrow \quad \underline{B} = \underline{m}_0 \underline{H}$$

Thus:

$$\nabla \times \underline{H} = \underline{j}_f \quad \text{“amperes law”}$$

$$\oint \underline{H} \cdot d\underline{l} = I_f$$

Energy in magnetic fields is thus:

$$U = \frac{1}{2} \underline{B} \cdot \underline{H}$$

For non-ferromagnetic materials $\underline{m} \sim 1$
 \underline{H} -field lines can be discontinuous:

$$\nabla \cdot \underline{H} = -\nabla \cdot \underline{M}$$

At boundaries:

$$B_{\text{perp}} \quad \text{continuous}$$

$$H_{\parallel} \quad \text{continuous}$$