An Application of the Fabry-Perot Etalon Arrangement

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This experiment was performed as a collaboration between S.Pike and J.Pearson

Abstract:

This experiment used the Fabry-Perot configuration of an interferometer: an etalon.

The etalon was initially used to determine the internal dimension of the spacing of the etalon – which was found to be $t = 2.3038 \pm 0.00029 \times 10^{-5}$ m. This was determined by a combination of measurement & the method of exact fractions.

Once the thickness of the etalon was determined, we produced predictions of Cauchy's constants – which are a characteristic of the glass prism used in measurements. To do this, we measured the refractive index, $m(l)$, of the glass prism.

The Cauchy constants determined were:

$$A = 1.6743 \pm 0.0005$$

$$B = 7.84 \pm 0.30 \times 10^{-15}$$

$$C = 5.78 \pm 0.39 \times 10^{-28}$$

1. Introduction[1],[2]

This report will outline the experimental procedure undertaken to measure the internal spacing within a Fabry-Perot etalon, and to derive the Cauchy constants for a particular glass prism.

As well as doing this, the report will begin by detailing the theory underlying the experiment, by explaining what an etalon is, and why it is of use in this context.

The report concludes with a section on the errors involved with the experiment, and how the experiment can be made more accurate.

1.1. Fabry- Perot Interferometer

Charles Fabry & Alfred Perot, in 1899, constructed the etalon-interferometer system. It is used to investigate, or "resolve" the hyperfine structure of spectral lines.

An interferometer, physically, is two parallel glass (or silica) plates: each with their inner surface coated in a reflective covering. Air is usually used to fill the gap. The plates can be moved apart to change functionality. See Fig 1.

The un-slivered outward facing surfaces of the bounding silica are created to have a slight wedge shape – by a few arc minutes – to reduce any interference patterns arising from reflection off these sides.

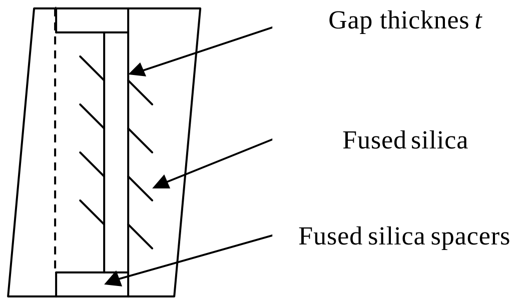


Fig 1. A Schematic diagram of an etalon.

When the plates of the interferometer are fixed, the configuration is known as a Fabry-Perot etalon. The slivered edges of the silica plates are separated by a thickness *t* – which was to be found along the course of the experiment, using the mathematical method of exact fractions.

Compared with a diffraction grating of (say) 5000lines/cm, the Fabry-Perot etalon has a resolving power of $\sim 10^6$ - where the grating has a power of ~8.

The reason for it having such a high resolution is due to the way in which it reflects the light within its boundaries: back and forth – each time the light bounces, some gets "transmitted" through the semi-silvered surface of the bounding glass/silica surfaces. These transmitted beams of light are then collected by a lens, which then get focused onto a point.

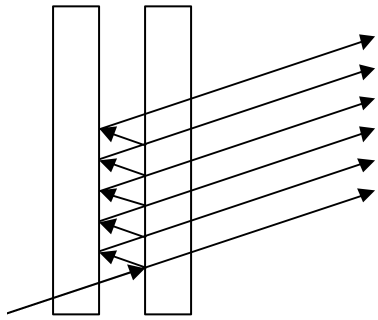


Fig 2. A diagram to show how an incident light ray is both transmitted & reflected by the internal surfaces of the etalon.

1.2. Method of Exact Fractions[3]

The method of exact fractions is a mathematical technique that allows distances to be measured, by measuring the fraction by which it exceeds an integer-number of wavelengths.

So, for the application to the etalon, we have:

$$2t = \left(n + \frac{g}{p} \right) l \qquad (1)$$

Where *n* is a number – whose value is known as the order of spectral lines, on axis. And the g/p term gives the fraction by which *2t* (the optical path) exceeds a whole number of wavelengths. Hence, *t* is the width of the etalon.

To find *t* we need at least three known spectral lines, to do the calculations reasonably accurately.

1.3. The Hyperfine Structure

The hyperfine structure of an optical spectrum arises from differences between energy levels within an energy state. This comes from a failure of the angular momentum/spin coupling within the atomic structure[4].

Usually, a discrete spectral pattern is seen when excited electrons "travel" between the discrete energy levels in atoms. These energy levels are dictated by certain quantum numbers, and every so often these quantum numbers arrange to give energy states which are incredibly close to each other.

For example, in hydrogen, the ground level is split into two separate states, which are separated by 5.9×10^{-6} eV [5] . Hence, if one has equipment which could resolve down to this scale, one would see two spectral lines. The so called "hyperfine structure" of an atom.

The Fabry-Perot etalon allows such resolution.

1.4. Edser-Butler Fringes

Now, the experimental setup was such that the spectral lines of a Mercury (Hg) lamp were superimposed over those of a white-light source. The spectral lines of the white light source are known as "Edser-Butler" fringes – EB fringes. The wavelengths of the visible Hg spectral lines were known – and can be found from various data sources[6] – and hence calibration of the EB fringes becomes possible; via the method of exact fractions.

1.5. Cauchy Constants

Refractive indices and wavelengths are related by the function[7]:

$$m(l) = A + \frac{B}{l^2} + \frac{C}{l^4} \qquad (2)$$

Where A, B & C are the Cauchy constants – a characteristic for the particular glass prism used; l the wavelength of the light; and m the refractive index of the glass, for a particular wavelength. The physical meaning of the constant A, is the refractive index approached as the wavelength increases to infinity.

2. Method

2. 1. Calibration

The telescope was focussed on a very distant object on the horizon – effectively now being focused on infinity. The equipment was then assembled as shown in Fig 3. without the prism, making sure that the white light was able to pass directly into the eye. The prism was placed onto the tray (see Fig 3.), and the angle adjusted so that the light still passed through the eyepiece into the eye. The light passes through the prism and is refracted differently depending on the wavelength of the light.

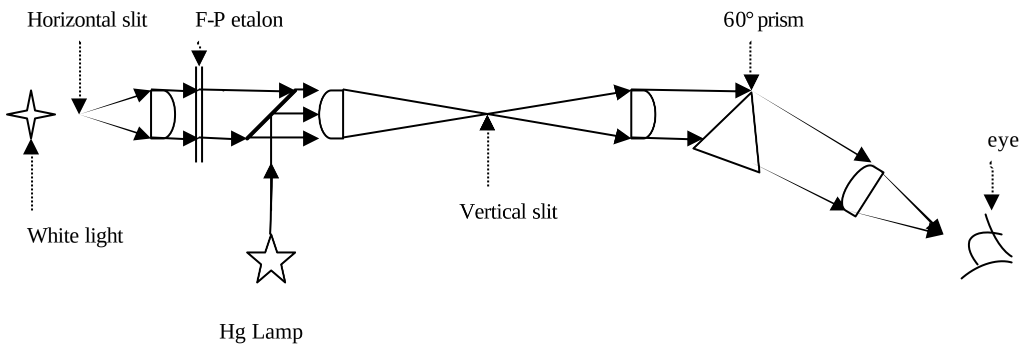


Fig 3. A schematic diagram of the apparatus, showing the positions of the white light source, etalon & mercury lamp.

2.2. Calculating n_0 and t

Gap thickness t, can be calculated by method of exact fractions. Equation (1) can be written in the form of (3).

Know, we know the wavelengths of the first four visible spectral line of Mercury (Hg); from data tables – see Table 1 in the Appendix.

n_0 is an integer corresponding to the refraction of the EB fringe that is before the first Hg fringe. P_m is the whole number of EB fringes between n_0 and the EB fringes before the corresponding Hg line. e_m is the fraction of the distance between 2 adjacent EB fringes that the Hg sits in. (see Fig 4 and equation (4).). The g -term accounts for any phase changes that occur during reflection inside the etalon.

$$2t - \frac{g_m l_m}{p} = (n_0 + P_m + e_m) l_m \qquad (3)$$

$$e_m = \frac{a}{b} \qquad (4)$$

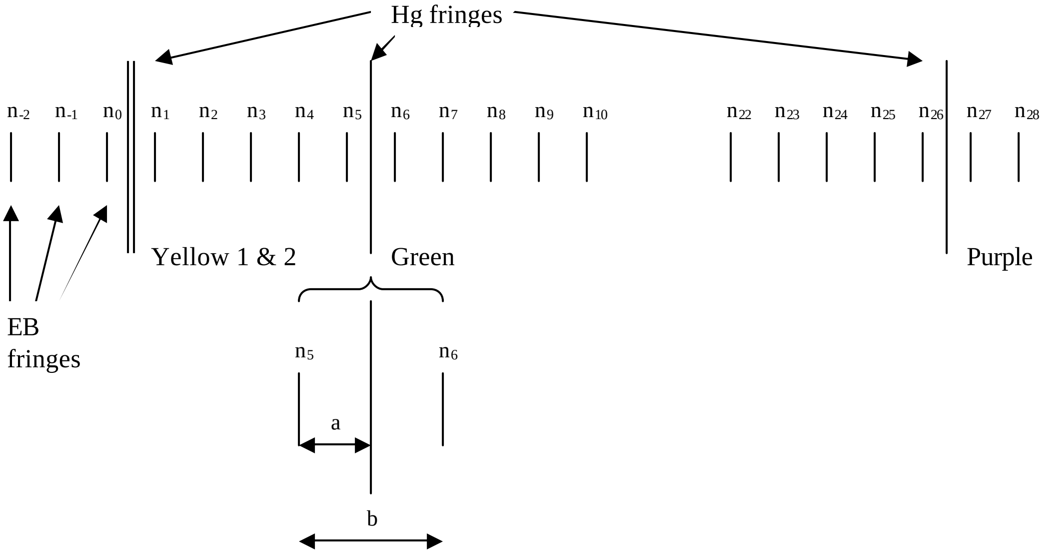


Fig 4. This schematic shows the relative positions of the spectral lines of Mercury (Hg) and the Edser-Butler fringes. Also shown is how the measurements for e_m were found.

The angle through which the light was refracted was measured for the n_0 Y_1, Y_2 ,n_1, n_1, n_5, G, n_6 ,n_{26}, P & n_{27} spectral lines. These values were used to find values of P_m and e_m for each Hg line. ? was calculated for each Hg line by plotting know wavelengths and their corresponding ? (Fig 5.), and using the graph to find a linear equation (5) relating ? to wavelength.

$$g(l) = 0.00166 l + 1.5 \qquad (5)$$

Equation (3) was written out for each Hg line. Combinations of which where used to solve for values for n_0 by simultaneous equations.

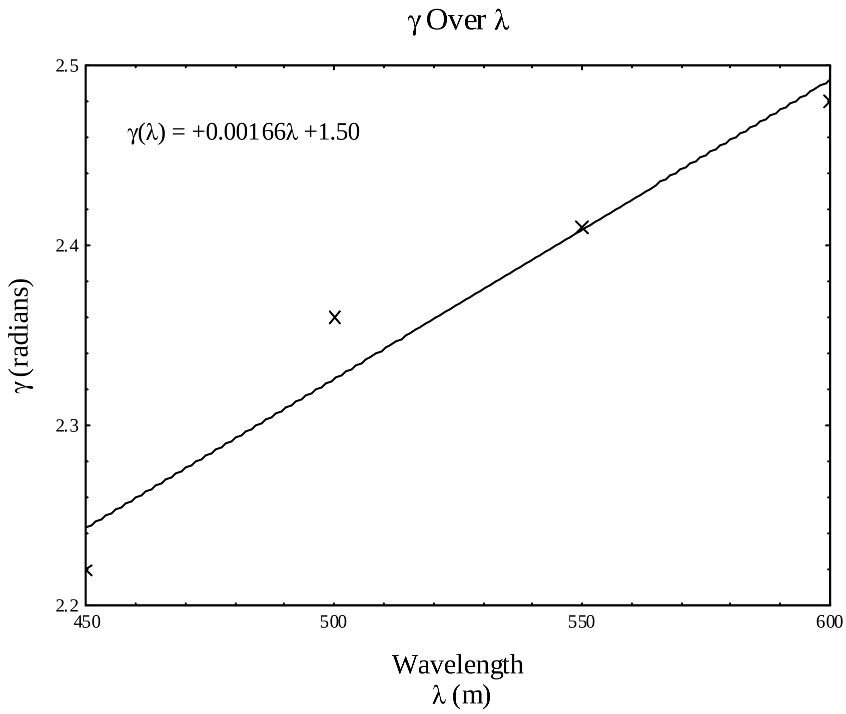


Fig 5. A graph to show the linear line of best fit, giving the phase change term as a function of wavelength.

The lines Y_1 and Y_2, G and Y_1, P and Y_1, and P and G were compared. As the distance between two Hg fringes increases, the errors on our readings for e_m are constant. This means that the error on n_0 decreases when two lines that are far apart are compared. With this in mind a graph of n_0 against separation was plotted, it was clear that the value of n_0 converged on 78 – see Fig 6.

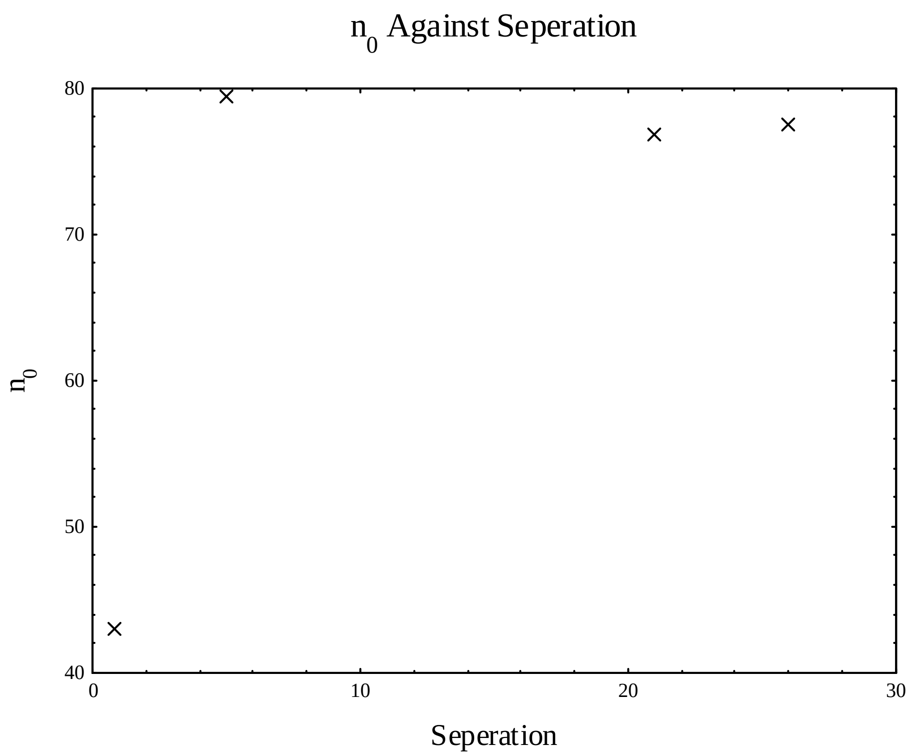


Fig 6. This shows a plot of n_0 against separation, where, upon close examination, one can see that the points will eventually converge on 78.

Using our result of $n_{0,}$ values of t were found for each Hg line, using equation (3). The average of these values was found to 2.3038×10^{-5} m, and the standard deviation was calculated. Hence:

$$t = 2.3038 \pm 0.00029 \times 10^{-5} \text{ m} \qquad (6)$$

3. Calculating the Cauchy Constants

3.1. Calibration

To investigate the refractive index, the prism has to be placed in a very specific way. First the prism was removed and a zero reading taken on the Vernier scale. The prism was placed back onto the 'tray' and was twisted whilst being followed by the eye piece until the image was seen to change direction. This position was known as the angle of minimum refraction, or deviation – and is denoted by d_m.

3.2. Finding Refractive Index and Wavelength

The angle of refraction for each visible EB fringe, from n_{-14} to n_{32} was found by finding the difference between the zero reading and the new angle through which the light was refracted. The refractive index was found using equation (7)[8], where d_m is the angle of refraction and a is 60°.

$$m = \frac{\sin\left(\frac{d_m + a}{2}\right)}{\sin\left(\frac{a}{2}\right)} \qquad (7)$$

The wavelength was found by combining equations (3) and (5) to create equation (8), which was solved to give two values of ? for each EB fringe. One of which was discarded as unphysical, due to the negative root of the quadratic.

$$l^2 0.00166 + (n_0 p + P_m p + 1.5)\, l - 2pt = 0 \qquad (8)$$

Refractive index m was plotted against $1/l^2$ and a quadratic curve fitted (Fig 7.).

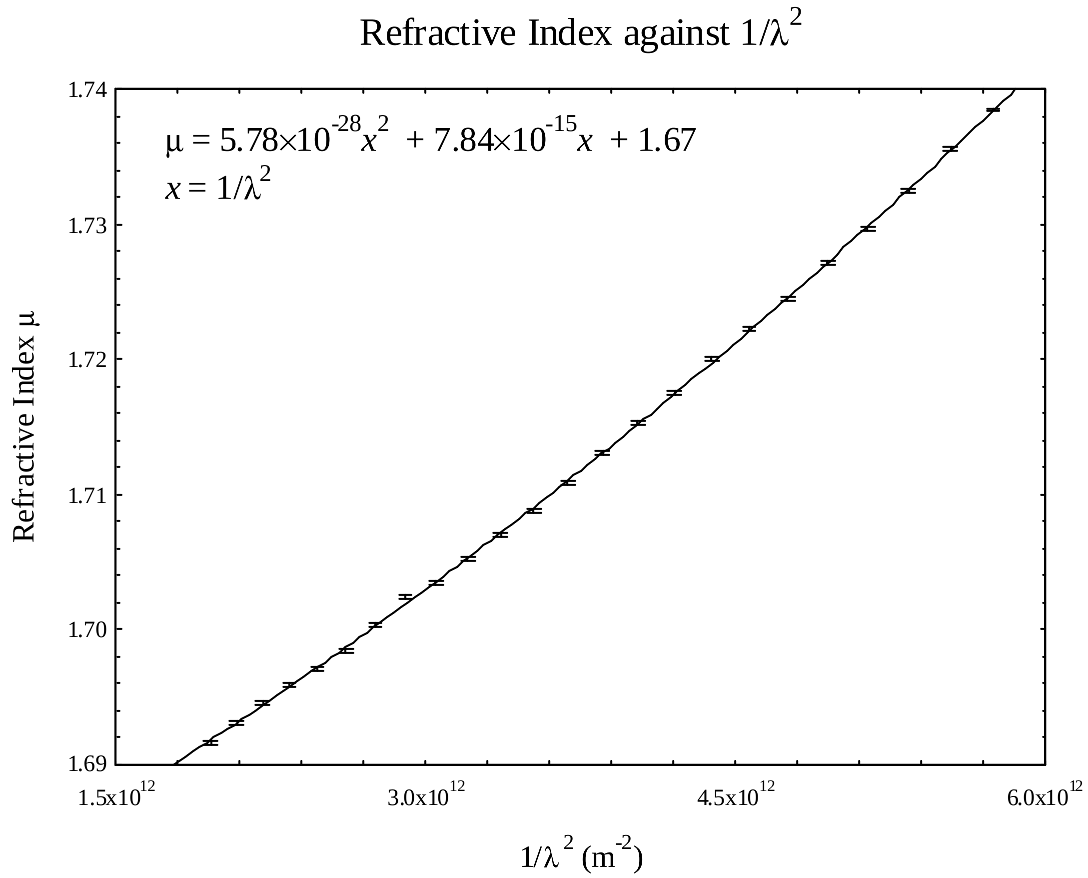


Fig 7. A graph of $1/?^2$ against the refractive index. Giving the Cauchy constants as the coefficients in the quadratic line of best fit.

The equation of the curve relates to Cauchy's formula (2),

$$m = A + \frac{B}{l^2} + \frac{C}{l^4} \qquad (2)$$

giving the values of the Cauchy constants to be:

$$\begin{aligned} A &= 1.6743 \pm 0.0005 \\ B &= 7.84 \pm 0.30 \times 10^{-15} \\ C &= 5.78 \pm 0.39 \times 10^{-28} \end{aligned} \qquad (9)$$

4. Error Analysis

The error on the minimum deviation angle d_m was decided to be ± 1.5 arc minutes. These errors arose from the fallibility of the human eye, when measuring a Vernier scale.

When this error was propagated through, via the MatLab computer package, the Cauchy constants and their errors are:

$$A = 1.6743 \pm 0.0005$$
$$B = 7.84 \pm 0.30 \times 10^{-15} \qquad (9)$$
$$C = 5.78 \pm 0.39 \times 10^{-28}$$

As the number of data points was 24, and there are 3 parameters in a quadratic fit, the number of degrees of freedom (*NDF*) is 24 – 3 = 21. For acceptable errors, a value of c^2 / NDF should be within the range,

$$1 - \sqrt{\frac{8}{NDF}} \leq \frac{c^2}{NDF} \leq 1 + \sqrt{\frac{8}{NDF}} \qquad (10)$$

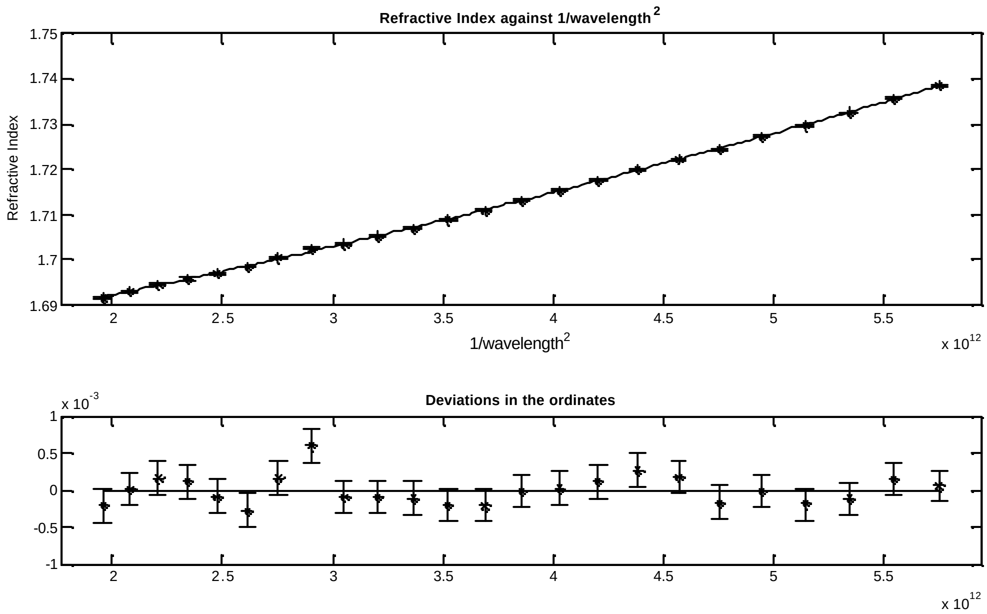
for a sample of 20 or more data points.

MatLab derived the value,

$$c^2 / NDF = 0.8161 \qquad (11)$$

which is within the acceptable range outlined above.

We then used MatLab to print a number of graphs, which are below, and show the deviation of the ordinates:



A	1.6743	+-	0.00054648
B	7.8389e-015	+-	3.039e-016
C	5.783e-028	+-	3.9402e-029
Chisq/(N-3)	0.82161		21 degrees of freedom

Fig 8. Printout from MatLab, with the values of the coefficients of the quadratic line of best fit, as well as their errors. Also showing the deviation of the ordinates from the line of best fit.

4.1. Error Reduction

As was previously mentioned, the main source of errors were the human eye, and its inability to measure Vernier scales to a high degree of accuracy. This can be improved by replacing the eye by some sort of electronic device, capable of measuring the scale to a higher degree of accuracy.

Also, the wavelength range measured could have been increased by decreasing the ambient light within the "dark room".

5. References

[1] F.Smith & J.Thomson; Optics, 2nd Edition; Wiley; p207

[2] E.Hecht; Optics, 4th Edition; Addison Wesley; p421-425

[3] R.Ditchburn; Light, 3rd Edition, Vol 1; Academic Press; p338-339

[4] F.Mandl; Quantum Mechanics, 1st Edition; Wiley; p147

[5] H.Young & R.Freedman; University Physics, 11th Edition; Pearson; p1566

[6] R.Weast et al; Handbook of Chemistry & Physics, 61st Edition; CRC Press; E274

[7] E.Hecht, Optics; 4th Edition; Addison Wesley; p85 – equation is of the form of, but changing the names of the constants

[8] 2nd Year Laboratory Script; Edser Butler Fringes

6. Appendix

Hg calibration line wavelength (Å)	Colour
5790.66	Yellow (*Y1*)
5769.60	Yellow (*Y2*)
5460.74	Green (*G*)
4358.33	Purple (*P*)

Table 1. Showing the wavelengths, in Angstroms, of the first four visible spectral lines of Hg [6]

?(nm)	? (radians)
600	2.48
550	2.41
500	2.36
450	2.22

Table 2. This table shows the given data[8], used to determine Fig 5.