

Investigations on the Polarisation Properties of Sucrose

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Abstract

The aim of this experiment was to investigate the polarisation properties of sucrose solutions. In particular, the separation of the right- and left-handed polarised components of an incident electromagnetic wave.

The experiment investigated the polarisation angle as functions of path length of the incident electromagnetic wave, concentration and temperature; for solutions of sucrose.

It was observed that polarisation angle was directly proportional to path length, concentration and temperature.

1. Introduction

This experiment exploited the polarisation properties of the sugar molecule sucrose.

The term “angle of polarisation” is defined as the angle by which the medium changes the angle of polarisation for an incident electromagnetic wave.

If the EM wave is plane-polarised at the initial boundary, the angle by which the medium changes the angle of polarisation, with the medium under known and documented conditions – the datum – is set as zero. This angle is measured at the point where the wave leaves the medium. Then the properties of the medium are changed, and thus the angle of polarisation which is measured, will be the angle for the medium under the controlled and documented conditions.

The methods of producing polarised light, and some properties of sucrose, will now be discussed; with the introduction section concluding with an outline of the experimental setup.

The report itself will continue with experimental data, and then conclude with an outline of how errors were decided, and a short description of how the experiment could be improved to be more accurate. With a proposal for further study described in section 2.2.

1.1 Polarisation^{[1],[2],[3]}

The light that is emitted from a sodium lamp, for example, is unpolarised. This means that the constituent electromagnetic wave contains vibrations in all planes, randomly.

However, when this unpolarised electromagnetic radiation (EMR) is passed through a polaroid – which is essentially a sheet of plastic with metal wires in one direction – the vibrations of the EMR which are in the direction of the wire are absorbed into the wire, thus giving the wires constituent electrons kinetic energy. This energy will eventually manifest itself as the heating up of the polaroid.

Now, these vibrations can be resolved into horizontal and vertical components, each mode of vibration with different amplitudes of vibration in a particular direction. Thus, if a particular direction is eliminated, by placing a polaroid in the path of incident EMR, then the EMR emitted from the polaroid will have vibrations only in the plane which is perpendicular to the wires of the polaroid.

This type of EMR is now denoted “plane polarised light”.

For example, if a wave is vibrating in the x - y -plane, and is incident upon a piece of polaroid which has wires in the x -direction only, then the wave which is emitted from the polaroid will only have vibrations with a y -component.

$$\underline{E}_I = E_{ox} \cos(kz - \omega t) \underline{\hat{i}} + E_{oy} \cos(kz - \omega t) \underline{\hat{j}} \quad (1)$$

Equation (1) represents an unpolarised plane wave, which is propagating in the positive z -direction. Where E_{ox}, E_{oy} are the respective amplitudes of the x - and y -components of the electric fields. With \hat{i}, \hat{j} as basis vectors aligned these axes.

If this wave is incident upon a polaroid aligned with the x -axis, then the resultant wave has the form:

$$\underline{E}_T = E_{oy} \cos(kz - \omega t) \hat{j} \quad (2)$$

Another type of polarisation is that of elliptical polarisation. Here, the EM wave is incident upon a *retarder*, which has the effect of adding a phase difference to the components of the EMR. So, if the plane wave described by equation (1), and when $E_{ox} = E_{oy}$, is incident upon a retarder which adds a phase change of \mathbf{d} to the y -component, then the transmitted wave is now given by:

$$\underline{E}_T = E_o \cos(kz - \omega t) \hat{i} + E_o \cos(kz - \omega t + \mathbf{d}) \hat{j} \quad (3)$$

If this phase change is $\mathbf{d} = -\mathbf{p}/2$, then the resultant wave is a circularly polarised wave, as $\cos(x - \mathbf{p}/2) = \sin(x)$:

$$\underline{E}_T = E_o \cos(kz - \omega t) \hat{i} + E_o \sin(kz - \omega t) \hat{j} \quad (4)$$

This wave is said to be circularly polarised, and will be observed to “corkscrew” through space, with the amplitudes remaining constant.

According to the convention which says that if a wave is seen to rotate in a clockwise sense, according to an observer towards whom the wave is moving, then (4) represents a right-circular polarised wave: R .

A phase change of $\mathbf{d} = +\mathbf{p}/2$ produces a wave of left-circular polarisation: L . Thus:

$$\underline{E}_T = E_o \cos(kz - \omega t) \hat{i} - E_o \sin(kz - \omega t) \hat{j} \quad (5)$$

Which will corkscrew in an anti-clockwise sense.

Elliptical polarisation arises from any phase difference other than $\mathbf{d} = \pm\mathbf{p}/2$. In this instance, the waves amplitude will change with time; thus tracing out an ellipse.

It is possible to change the sense of circular polarisation from R to L , say, by adding a phase difference to the circularly polarised light.

It is this phenomenon that was the subject of the experimental investigations.

1.2 Sucrose^[4]

This particular type of sugar has the chemical form of $C_{12}H_{21}O_{11}$, and is formed from glucose and fructose. It is postulated that due to the length of the sucrose molecule, a solution of sucrose will act like a retarder.

The specific properties of the sucrose retarder will depend on a number of parameters:

- Concentration of sucrose solution;
- Temperature of sucrose solution;
- Length of column of sucrose solution – path length;

These parameters will have the effect of adding different phase shifts to incident electromagnetic radiation.

1.3 Predictions

Previous to the experiment being performed, it was proposed that the relationship between the phase shift angle – the “angle of polarisation” – imposed by the solution, and the temperature of the solution; were linked proportionally, and possibly linearly.

It was supposed that this relationship would have come about by the temperature of the solution affecting the kinetic energy of the solutions molecules.

A molecule with high kinetic energy has a more random alignment than one with lower kinetic energy. And an increase of kinetic energy would be brought about by an increase in the temperature of the solution.

Thus, if the molecules are aligned more randomly, then the solution is more likely to have attained a higher phase shift parameter, hence increasing the angle of polarisation for incident electromagnetic radiation.

It was also proposed that the relationship between path length – the length of the column of sucrose solution that the incident electromagnetic radiation passed through – and the angle of polarisation would be proportional.

The reasoning behind this is similar to that for temperature.

If the wave has to travel through more molecules, then it will interact with more molecules, thus the wave will attain a higher angle of polarisation.

1.4 Experimental Setup^[5]

The equipment was arranged as Fig 1 shows. With the light source as a sodium lamp.

The purpose of having the split-field polariser is to introduce only plane polarised light into the column of solution – as well as providing some way of distinguishing between polarised and unpolarised light. The split-field polariser had half of its surface as a polariser aligned in one direction, and the other half aligned in another. The transmitted plane polarised light can be said to be made up of *R* and *L* circularly polarised light.

These R and L components will have different refractive indices, and will become separated when travelling through a medium. It is the separation of these components that will actually be measured, and denoted by “angle of polarisation”.

The viewing plate was simply a plate which could be rotated, with a hole to view incident light, with a protractor attached to take measurements.

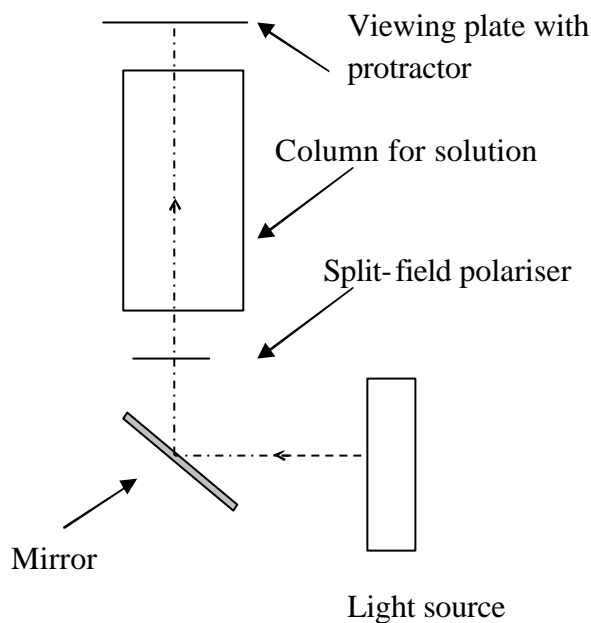


Fig 1. Schematic of apparatus

2. Experimental Procedures

A number of parameters of the sucrose solution were varied:

- Path length;
- Temperature of solution;
- Concentration of solution.

An experiment was conducted to determine the angle of specific rotation for the sucrose solution. This is defined as the angle of polarisation for a solution with concentration 1g/ml, with a path length of 10cm. The procedure used 1g/ml of solvent, and determined the specific rotation to be $40 \pm 1^\circ$.

The book value^[6] for specific rotation, for 1g/ml of solution, for sucrose is 66.4°

This section of the report will outline the procedures undertaken for each parameter, and summarising the results obtained.

2.1 Path Length

To investigate the relationship between the angle of polarisation of light, and the length of the path of the light through the sucrose solution; it was decided to take two sets of readings, for different concentrations, whilst keeping the concentration constant within each set of readings.

The concentrations used were:

8g sucrose and 100ml H₂O

25g sucrose and 100ml H₂O

All readings taken at room temperature, which was measured to be 22°C .

The results are tabulated in the appendix, and summarised by the following graph:

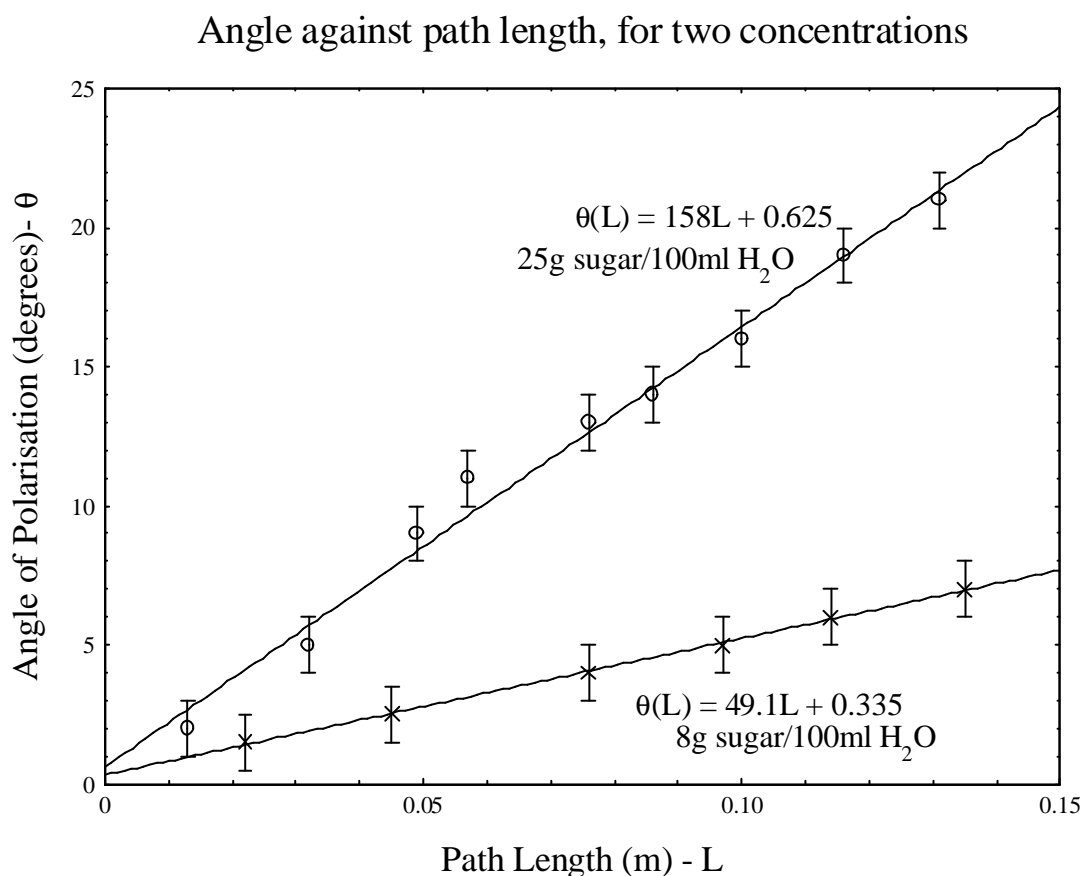


Fig 2. A graph of angle of polarisation against the path length, for two different concentrations of sucrose solution. A linear fit has been superimposed over the experimental data. With error bars corresponding to $\pm 1^\circ$ for angle measurements. A discussion on errors will follow.

It can be seen from Fig 2. that the prediction of the relationship between angle of polarisation and path length was correct: they are proportional and linear, within the errors decided upon. A discussion of the errors will follow.

It is also noted that the relationship between angle of polarisation and the path length is dependant upon the concentration of the solution. This is evident from the graph in Fig 2.

2.2 Temperature

The investigation between the temperature of sucrose solution, and the angle of polarisation used a constant concentration of 60g sucrose, and 60ml water.

Again, the results are summarised in Fig 2., and are tabulated in the appendix:

Angle of polarisation against temperature

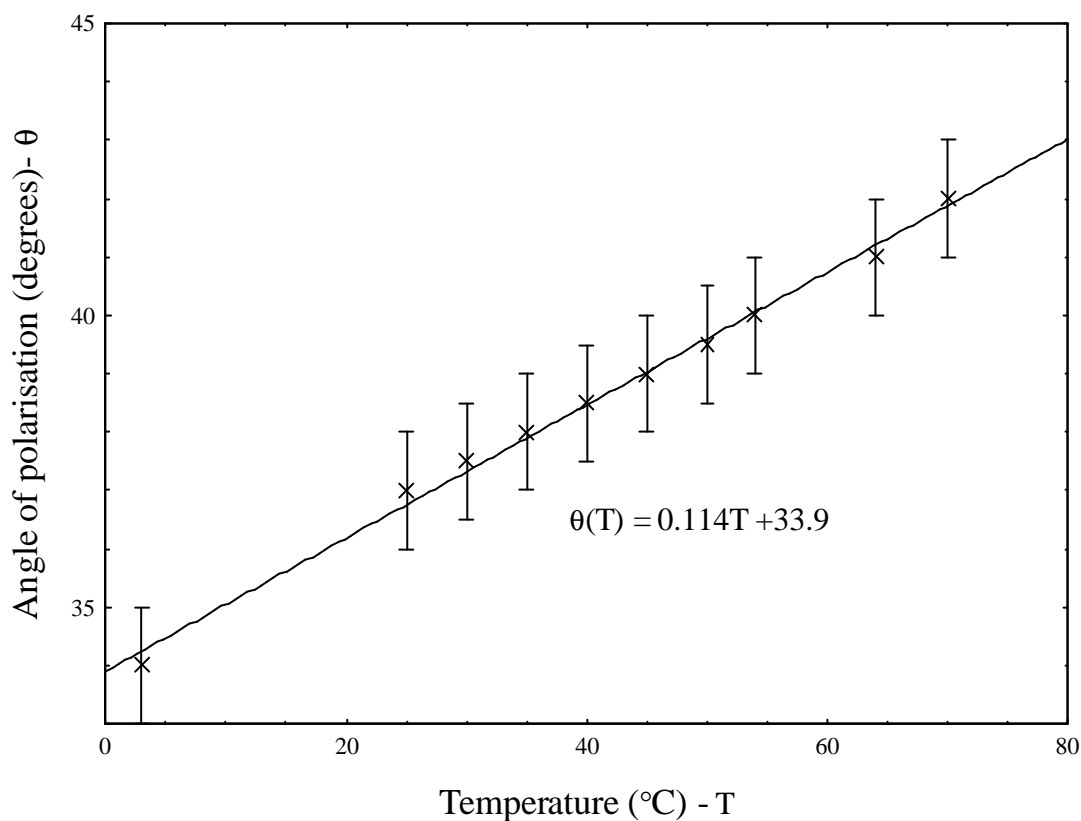


Fig 3. A graph showing the experimental data attained, when investigating the relationship between angle of polarisation and the temperature of the sucrose solution; for a constant concentration: 60g sucrose and 60ml H₂O. A linear fit has been superimposed over the experimental data. With error bars corresponding to $\pm 1^\circ$ for angle measurements. A discussion on errors will follow.

Again, one will notice that the initial prediction of a proportional relationship between temperature and angle of polarisation was correct.

It was supposed, that if the temperature was reduced until the sucrose solution was low enough for it to become solid, then the angle of polarisation would be a minimum. However, the equipment needed to do this extended experiment was not available, so could be not verified. This is an interesting proposal for further investigations.

2.3 Concentration

To investigate the relationship between the concentration of a sucrose solution and the angle of polarisation, a fixed volume of water was used (50ml), then incremented amounts of sucrose added. However, the addition of mass will increase

height of the column of solution – the path length. It has been previously shown that the angle of polarisation is dependant upon the path length.

The total change in path length was measured to be 4.5cm. A path length change of this magnitude would not alter the measurements sufficiently enough to become a problem. Hence, to a good approximation, the path length is a constant throughout this experiment. This was taken to be the initial path length: 7.6 cm.

Angle of polarisation against mass of sucrose, 50ml H₂O

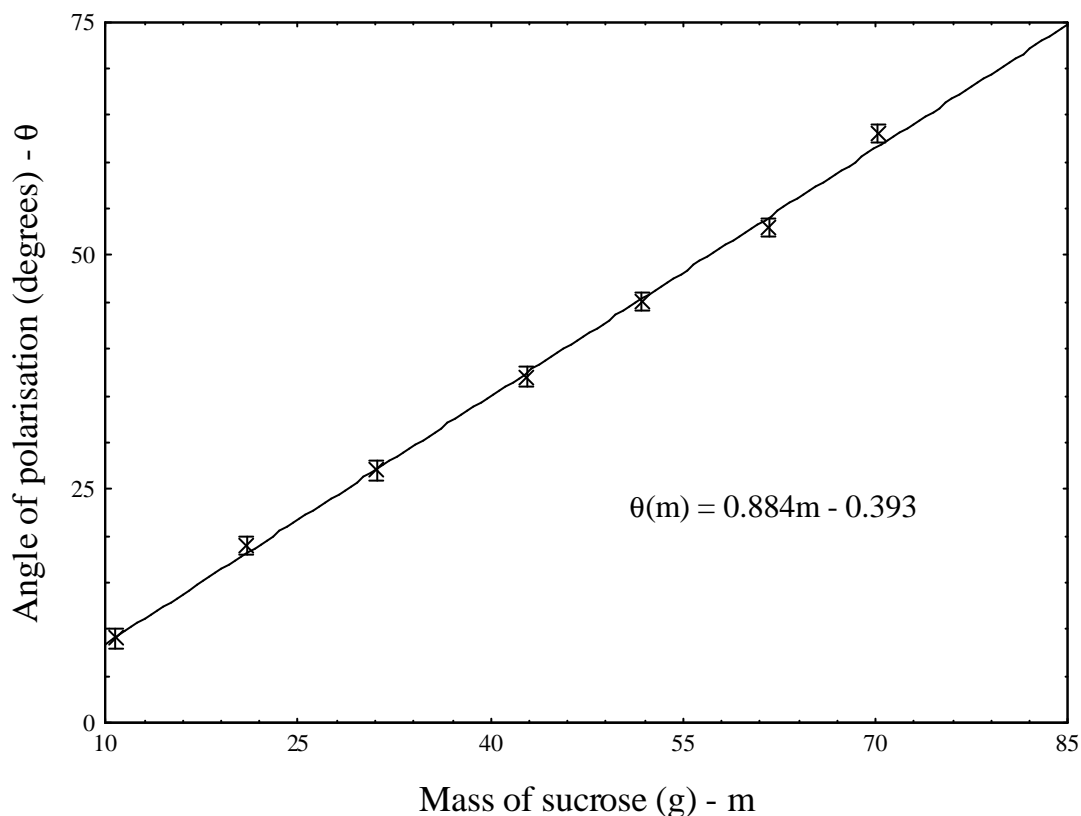


Fig 4. Experimental data for varied concentrations of a sucrose solution. A linear fit has been superimposed over the experimental data. With error bars corresponding to $\pm 1^\circ$ for angle measurements. A discussion on errors will follow.

Again, one can see that within the errors, the angle of polarisation and the concentration of sucrose solution are linearly proportional.

This experiment was carried out at room temperature: 22°C

3. Evaluation

Here, shall be presented the calculated chi-squared values, for both linear and quadratic fits, for the sucrose solutions. And comments on the initial linear fits shall follow.

3.1 Error Evaluation

The errors decided upon were as follows:

Parameter	Error
q	$\pm 1^\circ$
l	$\pm 0.0005\text{m}$

Table 1. Table summarising errors.

The relatively large error on measurements of the angle of polarisation q arose from the quality of the available equipment. To measure q , one first needed to calibrate the protractor to a “zero”, which carried an error, and then one needed to actually measure the angle of polarisation, which again carried an error. The combination of these errors were decided to be $\pm 1^\circ$.

The magnitude of this error will have a great effect on measurements in which there is not much change of the angle of polarisation.

The relatively smaller error on path length arose from the inherent human inaccuracies which arise when one measures scales by sight.

3.2 χ^2 Analysis

As the number of data points in all of the above experiments was relatively low, one may assume the relation,

$$\frac{NDF}{2} \leq \chi^2 \leq 2NDF \quad (6)$$

in evaluating the accuracy of experimental data.

Table 2. gives a summary of the χ^2 -analysis undertaken, with both linear and quadratic fits used as interesting comparisons.

The main method by which one could substantially reduce errors, is by taking more measurements, over a much larger range. More accurate equipment would also help.

Experiment	Proposed fit	NDF	χ^2
60g/60ml vary temp	Linear	8	0.2334
	Quadratic	7	0.1808
8g/100ml vary length	Linear	4	0.02897
	Quadratic	3	0.01051
25g/100ml vary length	Linear *	7	3.666
	Quadratic	6	2.394
Vary conc. 50ml	Linear *	5	4.181
	Quadratic *	4	3.354

Table 2. χ^2 -analysis summary.

One can quickly see how inaccurate the experiments were, and that no completely unambiguous conclusions can be drawn.

Only c^2 values for italicised fits provide useful conclusions, as it is only they which provide c^2 -values within the acceptable range given in (6). However, both linear and quadratic fits appear to be acceptable for the varied concentration investigations.

3.3 Property Functions

The experimental data which provided acceptable c^2 -values was used to determine the relationship between the relevant parameter, and the angle of polarisation. These relationships are summarised below:

25g sucrose, 100ml water; vary path length ℓ .

$$q_1(\ell) = 158.137\ell + 0.625 \quad (7)$$

Path length: 7.6cm; vary mass of sucrose m .

$$q_1(m) = 0.884m - 0.393 \quad (8)$$

$$q_2(m) = 0.001m^2 + 0.801m + 0.876 \quad (9)$$

Where $q_i(p)$ corresponds to the relation between the angle of polarisation q and the parameter p , to the i^{th} -order.

4. References

- [1] E.Hecht; Optics 4th Edition; Addison Wesley; p332
- [2] I.Grant, W.Phillips; Electromagnetism 2nd Edition; Wiley; p372
- [3] E.Hecht; Optics 4th Edition; Addison Wesley; p326
- [4] D.Voet, J.Voet; Biochemistry 2nd Edition; Wiley; p259
- [5] 2nd Year Laboratory Script; Optical Activity and the Faraday Effect
- [6] http://en.wikipedia.org/wiki/Specific_rotation

All graphs produced using EasyPlot, Spiral Software

5. Appendix

8g sugar/100ml H2O		25g sugar/100ml H2O	
Path length (m)	Angle (deg)	Path length (m)	Angle (deg)
0.022	1.5	0.013	2
0.045	2.5	0.032	5
0.076	4	0.049	9
0.097	5	0.057	11
0.114	6	0.076	13
0.135	7	0.086	14
		0.1	16
		0.116	19
		0.131	21

Table 3. Data obtained from the variation of path length, with two fixed concentrations of sucrose solution.

60g sugar/60ml H2O	
Temp (deg C)	Angle (deg)
70	42
64	41
54	40
50	39.5
45	39
40	38.5
35	38
30	37.5
25	37
3	34

Table 4. Data obtained the from variation of temperature, for a fixed concentration of sucrose solution.

Vary Conc - 50ml H2O		
Sugar (g)	Angle (deg)	Path length (m)
10.8	9	0.076
21	19	0.084
31.1	27	0.092
42.8	37	0.0103
51.8	45	0.0108
61.7	53	0.0115
70.2	63	0.0121

Table 5. Data obtained from the variation of mass of sugar. Note the column showing the change in path length. This change of path length has been ignore, justification given in section 2.3