Calculus of Variations:

$$I\{y(x)\} = \int_{x_1}^{x_2} f(x, y, y')dx \qquad \text{Where } y' = \frac{dy}{dx}$$

Gives the Euler equation:

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$ - The extremum condition.

Or, in another form:

$$f - y'\frac{df}{dy'} = 0$$ if f doesn't depend on x.

See that Euler's equation is like Lagrange's equation:

$$\frac{\partial f}{\partial y} = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) \leftrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

Principle of Least Action:

Now: $L = T - V$
$L(q_i, \dot{q}_i, t)$

For a moving body:

Initial: $t = t_1 \quad q_i(t_1) = q_i^1$
Final: $t = t_2 \quad q_i(t_2) = q_i^2$

Now, want to find a path that will minimise the action, A:

$$A = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t)dt = \int_{t_1}^{t_2} Ldt$$

$A\{q_i(t)\}$

Which gives the action as the Lagrange equation, but found from Euler's equation for least action:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

To minimise A, find the derivatives wrt to the amplitudes involved, and solve.