

Hamiltonian:

Now: $L(q_i, \dot{q}_i, t)$

And:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i}$$

- Lagrange Equation.

Generalised momenta:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

Thus, the Hamiltonian:

$$\begin{aligned} H &= \sum_i p_i \dot{q}_i - L \\ &= \sum_i \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L \end{aligned}$$

Hamilton's equations of motion:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \quad \frac{dH}{dt} = \frac{\partial H}{\partial t}$$

Note: the Hamiltonian should be in terms of the momenta *only*.