

Basic Relations:

Relativistic Energy/Momentum:

$$E^2 = p^2 c^2 + m^2 c^4$$

For photons:

$$E = pc$$

$$E = h\mathbf{n}$$

Hence:

$$p = \frac{h}{\mathbf{l}} \Rightarrow \mathbf{l} = \frac{h}{p}$$

Non-relativistic:

$$E = \frac{p^2}{2m} \quad \text{The KE}$$

Hence:

$$p = \sqrt{2mE}$$

$$\mathbf{l} = \frac{h}{\sqrt{2mE}}$$

Constructive interference in diffraction if:

$$d \sin \mathbf{q} = n\mathbf{l}$$

The Heisenberg uncertainty principle reads:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad \hbar = \frac{h}{2\mathbf{p}}$$

Now, $E = h\mathbf{n}$ and $\mathbf{w} = 2\mathbf{p}\mathbf{n}$. Therefore:

$$E = \hbar\mathbf{w}$$

Similarly:

$$k = \frac{2\mathbf{p}}{\mathbf{l}} = \frac{2\mathbf{p}p}{h} = \frac{p}{\hbar} \Rightarrow p = \hbar k$$

Hence, as $E = \frac{p^2}{2m}$, then:

$$\hbar\omega = \frac{\hbar^2 k^2}{2m}$$

The Schrödinger Equation:

The time dependant Schrödinger equation reads:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Suppose a solution is of the separable form:

$$\Psi(x,t) = \mathbf{y}(x)T(t)$$

Then:

$$\frac{\partial^2 \Psi}{\partial x^2} = T \frac{d^2 \mathbf{y}}{dx^2} \quad \frac{\partial \Psi}{\partial t} = \mathbf{y} \frac{dT}{dt}$$

Hence, the TDSE becomes:

$$-\frac{\hbar^2}{2m} T \frac{d^2 \mathbf{y}}{dx^2} + V(x)\mathbf{y}T = i\hbar \mathbf{y} \frac{dT}{dt}$$

Dividing by $\Psi = \mathbf{y}T$:

$$-\frac{\hbar^2}{2m\mathbf{y}} \frac{d^2 \mathbf{y}}{dx^2} + V(x) = \frac{i\hbar}{T} \frac{dT}{dt}$$

Which is only true if both sides are equal to a constant, E :

$$-\frac{\hbar^2}{2m\mathbf{y}} \frac{d^2 \mathbf{y}}{dx^2} + V(x) = E$$

AND: $\frac{i\hbar}{T} \frac{dT}{dt} = E$

The “time equation” can be solved for T :

$$\frac{1}{T} dT = \frac{E}{i\hbar} dt \Rightarrow \ln T = \frac{E}{i\hbar} t \Rightarrow T(t) = e^{\frac{Et}{i\hbar}} = e^{-\frac{iEt}{\hbar}} = e^{-i\omega t}$$

And the “space equation” becomes the time independent Schrödinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2 \mathbf{y}}{dx^2} + V(x)\mathbf{y} = E\mathbf{y}$$

And (notation) using operators this compresses to:

$$\hat{H}\mathbf{y} = E\mathbf{y}$$

Where:

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + V(\hat{x})$$

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x} \qquad \hat{T} = \frac{\hat{p}_x^2}{2m}$$

$$\hat{x} = x \qquad \therefore \hat{H} = \hat{T} + V(\hat{x})$$

Now, expectation values can be computed:

$$\langle p \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{p} \Psi dx \qquad \langle p^2 \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{p}^2 \Psi dx$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{x} \Psi dx \qquad \langle x^2 \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{x}^2 \Psi dx$$

Where Ψ^* is the complex conjugate of Ψ , which is usually a complex function. Hence the uncertainties can be found:

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2$$

Commutivity:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \begin{cases} = 0 \text{ comutate} \\ \neq 0 \text{ don't comutate} \end{cases}$$

1D Potential Wells:

$V = 0$ – in some cases, depends upon initial conditions.

Hence, TISE becomes:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \mathbf{y}}{\partial x^2} = E\mathbf{y}$$

Or:

$$\frac{\hbar^2}{2m} \frac{\partial^2 \mathbf{y}}{\partial x^2} + E\mathbf{y} = 0$$

Or:

$$\frac{\partial^2 \mathbf{y}}{\partial x^2} + k^2 \mathbf{y} = 0$$

With:

$$k^2 = \frac{2mE}{\hbar^2} \quad \Rightarrow \quad E = \frac{k^2 \hbar^2}{2m}$$

1D TDSE & TISE; Operators; 1D Potential Well

The solutions to the TISE $\hat{H}\mathbf{y} = E\mathbf{y}$ give eigenfunctions & eigenvalues:

$$\hat{H}\mathbf{y}_n = E_n\mathbf{y}_n$$