

Fourier Transforms:

A series:

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{inpx}{L}}$$

Where:

$$c_n = \frac{1}{2L} \int_{-L}^{+L} f(x) e^{-\frac{inpx}{L}} dx$$

Now, if we define:

$$k = \frac{n\mathbf{p}}{L}$$

$$\begin{aligned} g(k_n) &= \frac{L}{\mathbf{p}} c_n \\ &= \frac{1}{2\mathbf{p}} \int_{-L}^{+L} f(x) e^{-\frac{inpx}{L}} dx \end{aligned}$$

Hence:

$$\Delta k = \mathbf{p}/L \Rightarrow c_n = g(k) \Delta k$$

$$f(x) = \sum_{n=-\infty}^{+\infty} g(k_n) \Delta k e^{\frac{inpx}{L}}$$

Taking the limit as $\Delta k \rightarrow 0$:

$$\begin{aligned} f(x) &= \int_{-\infty}^{+\infty} g(k) e^{-ikx} dk \\ g(k) &= \frac{1}{2\mathbf{p}} \int_{-L}^{+L} f(x) e^{-ikx} dx \end{aligned}$$

$g(k)$ is the Fourier Transform of $f(x)$.

$f(x)$ is the inverse Fourier Transform of $g(k)$.