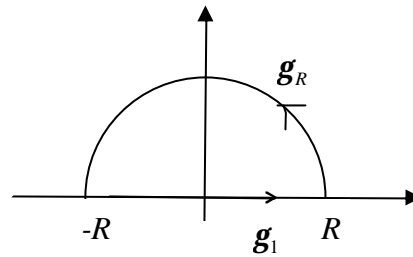


Jordan's lemma:

Consider the integral:

$$\oint_{\mathcal{g}} f(z) e^{imz} dz$$



Where:

$$\mathcal{g} = \mathcal{g}_1 + \mathcal{g}_R$$

Thus:

$$\begin{aligned} \oint_{\mathcal{g}} f(z) e^{imz} dz &= \int_{\mathcal{g}_1} f(z) e^{imz} dz + \int_{\mathcal{g}_R} f(z) e^{imz} dz \\ &= \int_{-R}^R f(z) e^{imz} dz + \int_{\mathcal{g}_R} f(z) e^{imz} dz \end{aligned}$$

Then, if the following conditions hold:

$$\begin{aligned} m &> 0 \\ \lim_{z \rightarrow \infty} |f(z)| &= 0 \\ f(z) e^{imz} &\text{ has only isolated, finite singularities} \end{aligned}$$

Then, as $R \rightarrow \infty$, that is to say, the semi-circular contour closes at infinity, then:

$$\oint_{\mathcal{g}} f(z) e^{imz} dz = \lim_{R \rightarrow \infty} \left\{ \int_{-R}^R f(z) e^{imz} dz + \int_{\mathcal{g}_R} f(z) e^{imz} dz \right\} = \int_{-\infty}^{\infty} f(z) e^{imz} dz$$

Notice that the semi-circular integral goes to zero. Hence:

$$\oint_{\mathcal{g}} f(z) e^{imz} dz = \int_{-\infty}^{\infty} f(z) e^{imz} dz = \int_{-\infty}^{\infty} f(x) e^{imx} dx$$

Which is just an integral along the real line.

Therefore, we can use this to evaluate real integrals by *Cauchy's residue theorem*:

$$\int_{-\infty}^{\infty} f(x) e^{imx} dx = \oint_{\mathcal{g}} f(z) e^{imz} dz = \frac{1}{2\pi i} \sum_k \text{Res} \{ f(z) e^{imz}; z = z_k \}$$

Notice that it is the sum over residues which are enclosed by the contour. Which is saying: sum over the residues due to singularities in the upper-half-plane.

This is all provided that the above conditions hold!