

Uniform stream:

Speed U at angle \mathbf{a} to the x -axis. Thus, the velocity vector is just:

$$\underline{u} = (U \cos \mathbf{a}, U \sin \mathbf{a}) = (u, v)$$

Now, the complex velocity is given by:

$$\frac{dw}{dz} = u - iv = U \cos \mathbf{a} - iU \sin \mathbf{a} = Ue^{-i\mathbf{a}}$$

Thus:

$$\frac{dw}{dz} = Ue^{-i\mathbf{a}} \Rightarrow dw = Ue^{-i\mathbf{a}} dz$$

$$\therefore \boxed{w(z) = Uze^{-i\mathbf{a}}}$$

Source:

At any radius, mass flux must be the same. The velocity is also purely radial.

$$2\mathbf{p}rU = m \quad m \text{ some constant}$$

$$\frac{dw}{dz} = u - iv = U \cos \mathbf{q} - U i \sin \mathbf{q} = Ue^{-i\mathbf{q}}$$

$$\therefore \frac{dw}{dz} = \frac{U}{e^{i\mathbf{q}}} = \frac{m}{2\mathbf{p}r e^{i\mathbf{q}}} = \frac{m}{2\mathbf{p}z}$$

$$\therefore w(z) = \int \frac{m}{2\mathbf{p}z} dz = \frac{m}{2\mathbf{p}} \log(z)$$

Thus, for a source of strength m , at $z = z_0$, the complex potential is:

$$\boxed{w(z) = \frac{m}{2\mathbf{p}} \log(z - z_0)} \quad \text{if } m < 0, \text{ then sink}$$

Vortex:

Here, the velocity is purely tangential. Again, the mass flux through any radius is constant, k say:

$$2\mathbf{p}rV = k$$

$$\underline{u} = V \hat{\mathbf{q}} = (-V \sin \mathbf{q}, V \cos \mathbf{q})$$

$$\frac{dw}{dz} = u - iv = -iV(\cos \mathbf{q} - i \sin \mathbf{q}) = -iVe^{-i\mathbf{q}}$$

$$\therefore w(z) = -\frac{ik}{2\mathbf{p}} \log(z - z_0)$$

k +ve... anti-clockwise
 k -ve... clockwise convention

Dipole:

Suppose a source & a sink very close together...

$$w(z) = \frac{m}{2\mathbf{p}} \log(z - \mathbf{d}e^{i\mathbf{a}}) - \frac{m}{2\mathbf{p}} \log(z)$$

$$= \frac{m}{2\mathbf{p}} \log\left(1 - \frac{\mathbf{d}e^{i\mathbf{a}}}{z}\right)$$

Now, let $\mathbf{d} \rightarrow 0$ & $\mathbf{d}m \equiv \mathbf{m}$ be constant.

Using the expansion:

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$$

Get:

$$w(z) = \frac{m}{2\mathbf{p}} \left(-\frac{\mathbf{d}e^{i\mathbf{a}}}{z} - \frac{\mathbf{d}^2 e^{2i\mathbf{a}}}{2z^2} - \dots \right)$$

$$= -\frac{\mathbf{m}}{2\mathbf{p}} \left(\frac{e^{i\mathbf{a}}}{z} + \frac{\mathbf{d}e^{2i\mathbf{a}}}{2z^2} + O(\mathbf{d}^2) \right)$$

\therefore as $\mathbf{d} \rightarrow 0$:

$$w(z) = -\frac{\mathbf{m}e^{i\mathbf{a}}}{2\mathbf{p}(z - z_0)}$$

for a dipole at $z = z_0$, inclined by \mathbf{a}

Flow in a corner:

$$w(z) = Az^m = Ar^m e^{im\mathbf{q}}$$