

## Method of Images

e.g.

$$w(z) = \frac{m}{2\mathbf{p}} \log(z-a) + \frac{m}{2\mathbf{p}} \log(z+a)$$

Is equivalent to a source at  $z = a$ , with a boundary on  $x = 0$ .

Mirror all sources/sinks/vortices in all “boundaries”

Remember that on streamlines:

$$\mathbf{y} = \text{Im}\{w(z)\} = \text{const}$$

Notation:  $\bar{f}(z)$  conjugates the function, not  $z$ .

Circle theorem:

If  $w = f(z)$  is a given flow, and if a circle is placed in the flow at the origin, with a radius  $|z| = a$ , then, under the assumption that  $f(z)$  is analytic inside & on the circle, the new potential is:

$$w(z) = f(z) + \bar{f}\left(\frac{a^2}{z}\right)$$

e.g.

1) flow due to a source  $m$  at  $z = z_0$ , around a circle  $|z| = a$ .

Source flow:  $f(z) = \frac{m}{2\mathbf{p}} \log(z - z_0)$

Conjugate function:  $\bar{f}(z) = \frac{m}{2\mathbf{p}} \log(z - \bar{z}_0)$

$$\begin{aligned} \therefore \bar{f}\left(\frac{a^2}{z}\right) &= \frac{m}{2\mathbf{p}} \log\left(\frac{a^2}{z} - \bar{z}_0\right) \\ &= \frac{m}{2\mathbf{p}} \log\left(\frac{a^2 - z\bar{z}_0}{z}\right) \\ &= \frac{m}{2\mathbf{p}} \log\left(-\bar{z}_0 - \frac{-\frac{a^2}{z_0} + z}{z}\right) \\ &= \frac{m}{2\mathbf{p}} \log\left(z - \frac{a^2}{\bar{z}_0}\right) - \frac{m}{2\mathbf{p}} \log(z) + \underbrace{\frac{m}{2\mathbf{p}} \log(-\bar{z}_0)}_{\text{const}} \end{aligned}$$

Thus, the final complex potential is:

$$\begin{aligned}
 w(z) &= f(z) + \bar{f}\left(\frac{a^2}{z}\right) \\
 &= \underbrace{\frac{m}{2\mathbf{p}} \log(z - z_0)}_{\text{original source}} + \underbrace{\frac{m}{2\mathbf{p}} \log\left(z - \frac{a^2}{\bar{z}_0}\right)}_{\text{image at } \frac{a^2}{\bar{z}_0}} - \underbrace{\frac{m}{2\mathbf{p}} \log(z)}_{\text{image at origin}}
 \end{aligned}$$

2) Uniform flow past a circle, with  $\mathbf{a} = 0$ .

$$f(z) = Uz \quad \bar{f}(z) = Uz \quad f\left(\frac{a^2}{z}\right) = \frac{Ua^2}{z}$$

Thus:  $w(z) = Uz + \frac{Ua^2}{z}$

$$= U \left( re^{iq} + \frac{a^2}{r} e^{-iq} \right)$$

Thus, the streamlines are on:

$$\mathbf{y} = \text{Im}\{w(z)\} = U \left( r \sin \mathbf{q} - \frac{a^2}{r} \sin \mathbf{q} \right) = U \sin \mathbf{q} \left( r - \frac{a^2}{r} \right)$$

What we can do with this now, is work out the velocity. In polars this is:

$$\begin{aligned}
 \underline{u} &= \left( \frac{1}{r} \frac{\partial \mathbf{y}}{\partial \mathbf{q}}, -\frac{\partial \mathbf{y}}{\partial r} \right) \\
 &= \left( U \left( 1 - \frac{a^2}{r^2} \right) \cos \mathbf{q}, -U \left( 1 + \frac{a^2}{r^2} \right) \sin \mathbf{q} \right)
 \end{aligned}$$

From this, can find the velocity at certain points, e.g. on the circle. Can also find the stagnation points.