Waveforms:

Plane waves:

$$E(\underline{r},t)=\underline{E}_0 e^{i(\underline{k}\cdot\underline{r}\pm wt)}$$

Wavefront: surface of constant phase… i.e. $f=\underline{k}\cdot\underline{r}\pm wt=const$

Spherical Waves:

$$E=\frac{A}{r}e^{\pm i(kr\pm wt)}$$

$$v_p=\frac{w}{k} \qquad v_g=\frac{dw}{dk} \qquad \text{“phase first”}$$

Spectra:

Now, as $E=E_0 e^{-iwt}$ is a solution, so is a linear superposition. Thus:

$$E=\sum_n A_n e^{-iw_n t}$$

Thus, can take this up to infinitesimals, and get the inverse Fourier Transform:

$$E(t)=\int_{-\infty}^{\infty} A_w(w)e^{-iwt}\,dw \qquad \text{Inverse FT}$$

$$A_w(w)=\frac{1}{2p}\int_{-\infty}^{\infty} E(t)e^{iwt}\,dt \qquad \text{FT}$$

Hence the spectrum of light is the Fourier Transform of the electric field. Similarly, for spatial:

$$E(x)=\int_{-\infty}^{\infty} A_k(k)e^{iwt}\,dk \qquad \text{Inverse FT}$$

$$A_k(k)=\frac{1}{2p}\int_{-\infty}^{\infty} E(x)e^{-iwt}\,dx \qquad \text{FT}$$

Huygen’s Wavelets:

Every point on a wavefront acts as a point source of secondary spherical waves; these waves interfere to produce the next wavefront.

Fermat’s Principle:

Light takes minimum time to travel between two points.

i.e. it minimises *L*: $L = \sum_i n_i \ell_i$

ℓ_i is the path length of a ray, in a material of refractive index n_i.

Polarisation States:

Suppose we have the ray:

$$\underline{E} = \hat{\underline{x}} E_x e^{i(kz-\omega t)} + \hat{\underline{y}} E_y e^{i(kz-\omega t+\phi)}$$

Then:

Linear polarisation:

$\phi = 0$ or π

Inphase antiphase

Circular polarisation:

$E_x = E_y$ $\phi = \left|\frac{\pi}{2}\right|$

$\phi = +\frac{\pi}{2}$ LH

$\phi = -\frac{\pi}{2}$ RH

Elliptical polarisation:

ϕ arbitrary

Polarisation by reflection/scattering:

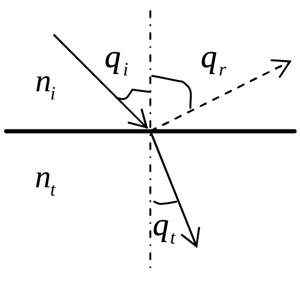
- from dipole emission… hence nothing emitted // to original E-field

Reflectivity is polarisation dependant:

If $\underline{E}$ is polarised // to place of incidence, then p-polarised.
If $\underline{E}$ is polarised perpendicularly to plane of incidence, then s-polarised.

Brewster Angle:

At certain angles of incidence, the angle of induced dipole will be // to direction of incidence…. Thus nothing reflected.



Zero amplitude reflected when:

$\theta_r + \theta_t = 90^\circ$

Thus: $\theta_i + \theta_t = 90$

Now, $\theta_i \equiv \theta_B$ "Brewster Angle"

Hence: $\theta_B + \theta_t = 90 \Rightarrow \theta_t = 90 - \theta_B$

Snell's law gives:

$$
\begin{aligned}
n_i \sin q_i &= n_t \sin q_t \\
n_i \sin q_B &= n_t \sin q_t \\
&= n_t \sin(90 - q_B) \\
&= n_t \cos q_B
\end{aligned}
$$

Thus: $\dfrac{\sin q_B}{\cos q_B} = \dfrac{n_t}{n_i}$

Hence, the Brewster Angle is given by:

$$q_B = \tan^{-1}\left(\frac{n_t}{n_i}\right)$$

Dichroism:

Some materials selectively absorb one polarisation state

E // to “long axis” does work in moving charge & is absorbed
E perp to “long axis” passes through
“long axis” is the axis of the polaroid… how molecules are arranged

Thus, transmitted field component:

$$E \cos q$$

Transmitted intensity:

$$I(q) = I_0 \cos^2(q)$$ Malus’ Law

Optical Anisotropy:

- materials with different permittivities in different directions
$e_x \neq e_y \neq e_z$

Hence, $\underline{\underline{e}}$ will be some matrix quantity

If: $e_x = e_y \neq e_z$ UNIaxial crystal
If: $e_x \neq e_y \neq e_z$ BIaxial crystal

Recall: $\underline{D} = e e_0 \underline{E}$, so now this is:

$$\underline{D} = e_0 \underline{\underline{e}}\,\underline{E}$$

Hence, $\nabla \cdot \underline{E} \neq 0$, thus have to rederive the wave equation:

$$\nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E} = me_0 \underline{\underline{e}} \ddot{\underline{E}}$$

For a plane wave $\underline{E} = \underline{E}_0 e^{i(\underline{k}\underline{r} - wt)}$, can show that this becomes:

$$-\underline{k}(\underline{k} \cdot \underline{E}) + k^2 \underline{E} = w^2 me_0 \underline{\underline{e}} \underline{E}$$

Now, the refractive index n_i of light which corresponds to travel in the direction in which the permittivity is e_i, is linked by $n_i^2 = e_i$. Thus, for the case of a uniaxial crystal:

$$e_x = e_y = n_o^2 \qquad\qquad e_z = n_e^2$$

Thus, two solutions to the wave equations, each with different refractive indices.

Birefringence: an anisotropic material with two refractive indices.

For the solution with refractive index n_o: ordinary ray "o-ray"
For the solution with refractive index n_e: extraordinary ray "e-ray"

The refractive index of the e-ray depends on q.
However, when $q = 0$, $n_o = n_e$ … which is defined as the "*optic axis*"…. When e-ray & o-ray become indistinguishable.

Note again:	if $e_x = e_y \neq e_z$	one optic axis	uniaxial
	if $e_x \neq e_y \neq e_z$	two optic axes	biaxial

e- & o-rays have the following properties:

$\underline{k} \cdot \underline{E}_o = 0$	o-ray perp to $\underline{k}$
$E_{o_z} = 0$	o-ray perp to optic axis
$\underline{E}_o \cdot \underline{E}_e = 0$	o-ray & e-ray are perpendicular to each other
$\underline{k} \cdot \underline{E}_e \neq= 0$	(unless $q = 0$, then $\underline{E}_o = \underline{E}_e$)

Now, $\underline{S} = \underline{E} \times \underline{B}$ (ish… with a factor of the permeability…)
Poynting

If: $\underline{E} = \underline{E}_0 e^{i(\underline{k} \cdot \underline{r} - wt)}$ $\qquad \underline{B} = \underline{B}_0 e^{i(\underline{k} \cdot \underline{r} - wt)}$
Then:

$$\nabla \times \underline{E} = \underline{k} \times \underline{E} \qquad \dot{\underline{B}} = -iw\underline{B} \qquad \underline{k} \times \underline{E} = -iw\underline{B}$$

from the Maxwell relation $\nabla \times \underline{E} = -\dot{\underline{B}}$

Hence, can rewrite the Poynting vector:

$$\underline{S} = \frac{1}{iw}\left(\underline{E}\times\underline{k}\times\underline{E}\right) = \frac{1}{iw}\left(\underline{k}\left(\underline{E}\cdot\underline{E}\right) - \underline{E}\left(\underline{E}\cdot\underline{k}\right)\right)$$

Thus, notice that for the o-ray, $\underline{E}\cdot\underline{k} = 0$ $\quad\therefore\quad$ $\underline{S} // \underline{k}$

Notice also that for the e-ray, $\underline{E}\cdot\underline{k} \neq 0$ $\quad\therefore\quad$ $\underline{S}$ not (generally) // to $\underline{k}$

(only for $q = 0$ again)

Now, when an $\underline{E}$-ray is incident upon a uniaxial crystal (not up optic axis), we get 2 beams… o-ray & e-ray… going in different directions… "double refraction"
Example of uniaxial crystal: calcite

Polarising beam splitter:

Suppose we stick two pieces of calcite together, which we can arrange so that the o-ray will undergo total internal reflection at the boundary of the two pieces, and that the e-ray passes through.

Remember that for TIR, the critical angle is dependant upon the refractive index, so the critical angle will thus be different for the o- & e-rays. $q_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$.

Hence, only one polarisation state leaves the "beam splitter".

Waveplates:

"fast"…	$E_{//}$ lags $E_{\perp}$	by d	$n_{\perp} > n_{//}$
"slow"…	$E_{\perp}$ lags $E_{//}$	by d	$n_{//} > n_{\perp}$

Halfwaveplate… $\quad d = p$

Quaterwaveplate… $\quad d = p/2$

- used to flip polarisation direction about optic axis.

Faraday Effect:

Plane of polarisation is rotated as light passes through a material in the presence of a $\underline{B}$-field. Rotation angle is given by:

$b = VBd$ $\qquad$ V = "Verdet's constant" $\qquad$ B = mag.field strength

d = path length through material

rotation is independent of direction, so, if reflected back, then $b' = 2b$