Second Sound in Liquid Helium

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Abstract

We measured the dependance of the speed of second sound in liquid Helium-4, on the temperature range $1.377 \leq T \leq 2.150$K, in a perspex cylindrical resonance cavity, with thermal contraction effects being considered. The dependance of viscosity on temperature is also investigated, using a decay time method, and coincidences of events investigated.

Introduction We use liquid Helium-4, below its lambda-transition temperature into a super-fluid: $T_\lambda = 2.16$K. For $T < T_\lambda$, Helium-4 is known as He II. We use a two-fluid model of He II, where the two parts act as normal- and super-fluid. These two parts can be thought of as being quantum super-positions; one is unable to classify one bit of the fluid as being a member of a particular part. If the total fluid density is ρ, then ρ at $T = 0$K is ρ_s: all fluid is super-fliud; and ρ at T_λ is ρ_n: all fluid is normal-fluid. This convention for refering to super- and normal-fluid via subscripts will be continued. An important part of this model, is that He II is modeled as being incompressible $\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n = 0$.
The model is such that the super-fluid component is unable to carry heat, only the normal-fluid component. Thus, if we induce small changes in temperature T^* in He II, we are able to setup standing temperature waves, with the normal- and super-fluid components oscillating in anti-phase. This is called second sound. He II has the property that it has extremely high thermal conductivity, and is hence virtually impossible to set up a temperature gradient across a sample. It can be derived (in 1D), that we have the wave equation

$$\frac{\partial^2 T^*}{\partial t^2} = \frac{T_0 S_0^2 \rho_s}{C_0 \rho_n} \frac{\partial^2 T^*}{\partial x^2}, \tag{1}$$

where T_0, S_0, C_0 are the ambient temperature, entropy and heat capacity [1]. This gives a general expression for the speed of such waves, where we may look up relevant parameters. Second sound speed is denoted u_2. Notice that u_2 is dependant upon there being super-fluid, and will thus be zero when $\rho_s = 0$; i.e for all $T \geq T_\lambda$.

Experimental Setup This experiment was based around the cryogenic cooling of Helium-4, where a blanket cryostat is filled with liquid Nitrogen. The systems pressure is controlled using a vacuum pump, thus the overall temperature can be varied, and measured using an oil manometer.

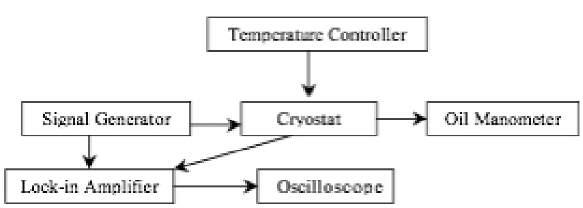

Figure 1: Block diagram of equipment. The lock-in amplifier acts to clean away any noise picked up inside the cryostat, by comparing a received signal with a reference from the driving signal, and averaging over a set time period.

The temperature of the liquid helium was fine-tuned using a Wheatstone bridge, with one of its 4 component reisistors within the liquid He, and has a temperature dependent resistance R. Another of its resistors is a variable resistor, and it is the value of this resistor we change. A temperature is fixed when the bridge is balanced across its arms. There is a resonance cavity immersed inside the He: a polished perspex cylinder, with epoxy endcaps. Halfway down the cavity are two small holes, otherwise the cavity is closed. Thin carbon films cover the inside face of both endcaps; where opposing films act as the transmitter and receiver for the temperature wave. There is a connection from an AC-signal generator to the transmitter (via a reference port on a lock-in amplifier), and another connection from the receiver to the lock-in amplifier. Schematics of apparatus used are in Figs(1) & (2).

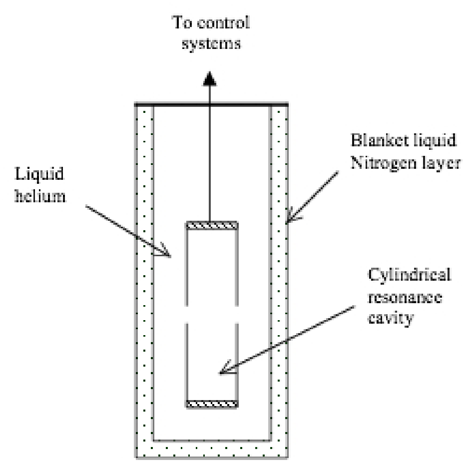

Figure 2: Cutaway of cryostat, showing resonance cavity. Where 'control' is referring to above input/output systems. The epoxy endcaps are shown with hashing. NOT TO SCALE.

This setup allowed a driving voltage ($V_0 = \frac{1}{2}V_{pp}$) and frequency f to be sent into the He-filled cavity from the signal generator to the transmitter, where any response is picked up by the receiver, and visualised using an oscilloscope. The transmitter is driven with a signal $V = V_0 \sin \omega_0 t$. Note $\omega_0 = 2\pi f$. Upon calculating the power input, $P = \frac{V^2}{R} = \frac{V_{pp}^2}{8R}(1 + \cos 2\omega_0 t)$, we notice that there is a constant DC-component to the input power. This component has the effect

of constantly heating the He II. The normal fluid component within the two-fluid model has the role of carrying this heat out, via the holes in the resonator: as normal fluid flows out, more fluid flows in. As input power is increased, this increases the rate at which fluid flows into the resonator, and induces vortices in the superfluid, which has the effect of attenuating motion of the fluid inside the resonator. Investigation of amplitude response with input power resulted in an ideal power of $P \approx 8$mW, where power-amplitude response is linear.
If there is a standing wave in a cavity of length L, with the requirement that we have antinodes at the ends (due to the huge differences in thermal conductivities of the epoxy end-caps & that of the fluid), we have the simple relation for the wavespeed u_2, resonance frequency f, and axial harmonic n: $u_2 = \frac{4Lf}{n}$.

Viscosity We measure the viscosity η of liquid helium using a decay time method. We setup a resonance in the cavity, and remove the signal generator input feed. We assume that the resonance width Δf and resonant frequency f are unaffected by this procedure. We capture the decay of a resonance via an oscilloscope printout, and thus gain a value for the decay time of a resonance. The decay time τ is directly proportional to the width of the resonance via $\Delta f = \frac{1}{2\pi\tau}$.
Zino'eva's work [2] supplies a relation for the viscosity,

$$\eta = \left(\frac{\Delta f}{\sqrt{f}}\right)^2 \frac{a^2\rho^2\pi\rho_n}{\rho_s^2}, \qquad (2)$$

where a is the radius of the resonance cavity. The relative densities can be looked up for a particular temperature [3].

Results On temperature range $1.377 \leq T \leq 2.003$K, an oil manometer was sufficient to directly measure temperatures. However, for $T > 2.003$K, a calibration between known temperatures and balanced resistances had to be used; so that from a balanced resistance, it is possible to deduce the temperature from an extrapolated fit.
There are holes halfway down the cavity; hence, even axial harmonics (n even) are proposed to be affected, as they have temperature antinodes at the holes, and thus radiate. This effect has been observed, and noted that all even harmonics have a higher normalised resonant frequency, with lower relative amplitude response. It is for this reason that it was chosen to use the $n = 5$ harmonic for all subsequent measurements.
Upon investigation of u_2 upon T, we find a dependance graphed in Fig(3). We note that there is a maximum speed $u_2 = 20.181 \pm 0.126\text{ms}^{-1}$ at $T = 1.650 \pm 0.003$K. This turning point marks the position where u_2 is independant of T and coincides with the minimum η of He II, 1.26×10^{-6}Pas, from [3].
Data obtained for the viscosity is shown in Fig(4).
From manufacturer schematics, we find $L_0 = 80.00 \pm 0.01$mm and $a = 6.00 \pm 0.01$mm; however, if thermal contraction effects are considered: $L = 78.33 \pm 0.48$mm, via [4].

Analysis & Conclusion All viscosities found are too high, relative to book values [3]. This may be due to all damping effects being attributed to viscosity, when this isn't necessarily the case. Zinov'eva briefly suggests that thermal conduction losses are an order of magnitude greater than those due to viscosity [2]. We have also assumed that Δf and f stay constant throughout the decay, when again, this may not be the case.

We find book values for the maximum of u_2 to be 20.4ms^{-1}, at $T = 1.6537$K [3]. This temperature is within our above errors, but the speed is not. This is probably due to innacurate thermal contraction data at such low temperatures.

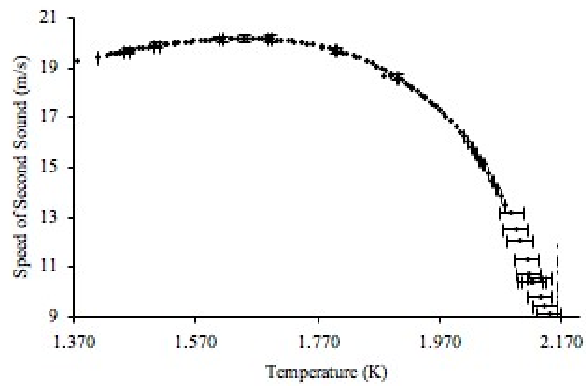

Figure 3: Plot showing dependance of speed of second sound u_2 with temperature T. The dashed line indicates $T = T_\lambda$.

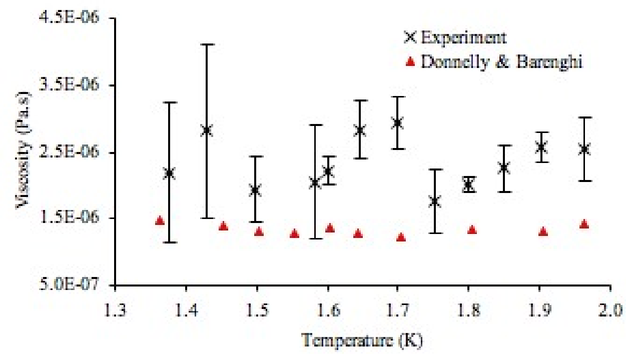

Figure 4: Plot showing dependance of viscosity upon temperature. Triangular points are book values, from Donnelly & Barenghi [3]

To conclude, we find that high power input attenuates second sound, supposed to be due to superfluid vortices. We have shown how the speed of second sound varies with temperature, and that the temperature at which u_2 is maximum agrees with book values. However, experimental data for η is of a very low quality.

References

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